

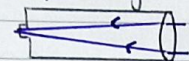
Astrophysics

Lecture 1

Astrophysics is the oldest science - in Religion, mythology, astrology, and then science.

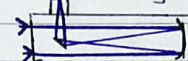
Telescopes.

Refracting



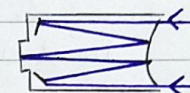
expensive
colour issues

Reflecting



cheaper
blurring

Catadioptric



Best
expensive

Aperture has diameter D , and when you look at stars they have rings around them from diffraction



Diffraction limit - minimal angle separation of 2 stars so they can be distinguished by telescope looks like  in a scope.

$$\Delta\theta = \frac{1.22 \lambda}{D}$$

Seeing.

Starlight through the layered atmosphere makes it look different, and placed differently aka Twinkling.
degrades diffraction limits from 0.5 to 2 arcseconds.

How do we beat it?

Adaptive Optics - computer calculating how crap weather is and changes scope optics to beat it.

Lasers

- see how the laser light changes and deforms

Just go into Space!

Lecture 2

Blackbody Radiation

Emits all EM Radiation absorbed, and absorbs all radiation incident.

$$L = A \sigma T^4$$

$$W = \lambda T$$

$$\text{from flux} \quad F = \frac{L}{A} \text{ or } \frac{L}{4\pi d^2}$$

Planck Radiation formulae:

from cosmology x

Energy per unit volume per unit frequency

or

wavelength

$$S_\nu = \frac{8\pi h}{3} \frac{\nu^3}{e^{h\nu/kT} - 1}$$

$$S_\lambda = \frac{8\pi h c}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Magnitudes

Made by Hipparchus - 1 is brightest to 6 dimmest.
Galileo saw fainter than 6 $\Rightarrow$ telescope invention.

Magnitude is logarithmic

A change in 5 magnitudes = 100x brightness change

$\therefore$ 1 mag change is $100^{1/5}$ change = 2.512

For 2 stars w/ magnitudes m_1 & m_2

Brightness ratio: $100^{(m_1 - m_2)/5}$

Flux equation: $\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}$

Mence: $-2.5 \log_{10} \left(\frac{F_2}{F_1} \right) = m_1 - m_2$

and

$$m_1 - m_2 = 5 \log_{10} \frac{d_1}{d_2}$$

Magnitudes are all ratios
(α Vega)
So we use α Vega as
a basic standard
with mag 0

Apparent magnitude - brightness from Earth (m)

Absolute magnitude - brightness from 10 pc (M)

$$m - M = (5 \log_{10}(d) - 5)$$

Filters

limits wavelengths observed to make better observations

Bandpass ($\Delta\lambda$) = bandwidth and has central wavelength λ_0 .

Photometry is the practice of measuring a
star's brightness/flux. Nowadays, it
is common to use CCD cameras to

measure the $\#$ γ from a star. Adding pixel values and
subtracting background radiation gives us the intensity.
This needs multiple steps + comparisons + pre-calibration.

$$1 \text{ angstrom} = 10^{-10} \text{ m}$$

Spectroscopy is splitting the star's light into a continuum. It
shows lines, which shows: Elemental/molecular composition,
temperature & pressure, rotation, magnetic field strength, velocity,
mass loss, etc...

$$\text{Resolution } R = \frac{\lambda}{\Delta\lambda}$$

Lecture 3

The Sun

Stats

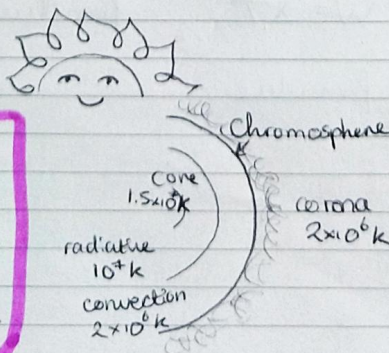
1.5×10^6 km away

$\theta = 1/2^\circ$

2×10^{30} kg = $M_\odot$

$T_{\text{eff}} = 5770$ K

74% H, 24% He, 2%



The Core

- ° Thermonuclear reactions $\approx 0.25 R_{\odot}$
- ° $T \approx 1.5 \times 10^7 \text{ K}$ $\rho \approx 150,000 \text{ kg m}^{-3}$
- ° Nuclear burning just about completely stops after $0.25 R_{\odot}$.

Radiative Zone

- ° $0.25 - 0.7 R_{\odot}$ ($0.7 R_{\odot}$ = Interface layer or Tachocline)
- ° Energy radiates outward - photon scattering - very slow (10^5 yrs)
- ° ρ is from 20,000 (centremost) to 200 kg m^{-3} .
- ° T falls from 7×10^6 to 2×10^6

Tachocline - between Radiative & convective.

- ° Magnetic field may be generated here by shearing (changes in fluid velocity)

Convective Zone

- ° $0.7 R_{\odot} - 1 R_{\odot}$
- ° $T = 2 \times 10^6 \text{ K}$ (cold enough for "metals" to have their electrons) photons from Radiative zone can't get past, traps heat
- ° Photons make fluid unstable and boils $\Rightarrow$ convection current.
- ° Surface $T \approx 5770 \text{ K}$ $\rho \approx 0.0002 \text{ kg m}^{-3}$

The Photosphere

- ° Visible surface, 100 km thick, non uniform intensity.

Sunspots



influx

Strong magnetic fields

granules

convection cells & 5 mins only

$\approx 1000 \text{ km}$



Faculae

light areas of concentrated magnetic field.

Chromosphere

$T \approx 20,000\text{ K}$ - gas strongly in H α line (6563 Angstroms)

- In H α can see chromospheric network, plages, filaments, prominences, spicules.

Transition Region

- Rapid temp change, hydrogen completely ionised, emission mostly of C IV, O IV, Si IV (UV mostly)

Corona

- $T \sim 10^6\text{ K}$, see it w/ eclipses!
- Emission mostly of Ca, Fe. H, He, N, C, O completely ionised & stripped.
- Can see loops, plumes, streamers, holes! Use X-ray!

Solar Phenomena

Limb Darkening

Centre of Sun is brighter/yellower than edges of Sun (also redder).

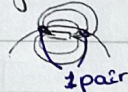
Looking at photosphere only, no core, no direct γ , very cool!



Looking at hotter denser, + core, very bright!

Sunspots

- Cools down to $\approx 3,800\text{ K}$. Intense B-field that interferes/halts convection in that area (hence it cools) goes to 0.1 T.
- Usually has a group of sunspots in an active region.



Prominences or filaments

- Streams of chromospheric gas. Couple types:

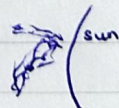
Quiescent - long lived (weeks) curtains slowly falling from corona $\rightarrow$ Chromosphere

Active - few hours & very loopy and can become solar flare.



Flares

- Small ones happen a few times an hour or every few hours one large one.
- 10k-300k km Smin to rise 20+ mins to fall. $\approx 10^6\text{ K}$. Energies upto 10^{30} J
- They eject energy + cosmic rays.



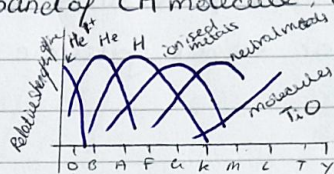
Solar Wind

Solar gravity isn't enough to hold high energy particles. Has to go into space through coronal holes or solar flares. Sun loses $\approx 10^{13}\text{ Mo/y}$. $v_0 = 300\text{ km/s}$ $v_{\text{Earth}} = 400\text{ km/s}$

Stellar Classification

Lecture 4

Harvard System OBAFGKM-LTY - mostly based on hydrogen spectral lines. Most important ones are Balmer series. They also used ionised He, Fe, Ca, H α , K, G band of CH molecule, and others.



Strongest lines

(almost accurate to scale)

also has
9-1 star system
between.

class

It is a temperature classification system.

Spectral differences primarily reflect differences in surface temperature.

- O - 50,000 K $\hat{a}$ no Hydrogen lines. Strong He⁺
- B - 20,000 K $\hat{a}$ weak hydrogen, strong neutral He
- A - 10,000 K strong H, weak Ca⁺
- F - 7,500 K strong Ca⁺, weak H,
- G - 6,000 K strong metals...

Yerkes (MKK) system estimate surface gravity $g = \frac{GM}{R^2}$ to define Giants Dwarves.

Ia luminous supergiant

Ib less luminous $\uparrow$

II luminous giants

III Giants

IV subgiants

V MS dwarf

VI Subdwarf.

dwarves have small R so g is higher.

dwarfs have higher pressure + surface density.

under high pressure atoms collide more frequently,

collisions shorten lifetime of excited state $\hat{o}$ lines broaden

normal

dwarf.

Special Mentions

WR Wolf-Rayet, He mostly, fast dying w/ large solar wind.

C Carbon, Red (Super)giants near death w/ excess carbon

D white dwarf (subclasses ABQZCX)

LTY Brown dwarf, failures!

Populations

Population 1 stars, young stars in spiral arms (like sun), in open clusters, ~2% metals. The Pleiades is an example.

Population 2 stars, old stars, first stars with only ~0.01% metals. Centre of galaxy or elliptic galaxies, globular clusters.

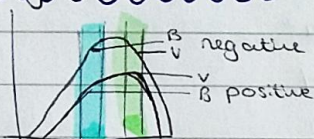
Colour

Difference between the brightness of stars in 2 filters. Filters U, B, V, R, I for example B-V colour = mag from B - mag from V. ← colour index

$$CI = m_{\lambda_1} - m_{\lambda_2}$$

You can use U-B, B-V, B-R, V-R, R-I, it all works!

Why is it important?



Hotter stars have more negative CIs.

better for observational astronomer

You can replace Temp with CI in a HR diagram! $x = CI$ like B-V & $y = \text{mag}$ gives a colour-magnitude diagram.

If you can observe the $CI^{(m)}$, you can find M using a HR diagram and calculate distance with $m - M = 5 \log d - 5$. This method is spectroscopic parallax.

Lecture 5

Stellar Structure

Hydrostatic Equilibrium.

2 main forces: Gravity (contraction) & Pressure (expansion) when there's perfect balance between the 2, we have hydrostatic equilibrium.

$$\frac{dP}{dr} = -\rho r \left(\frac{GM}{R^2} \right) = -g \rho r$$

change of pressure with radius.

Surface gravity (when distance $r = R$)

don't actually multiply r !

The Sun's central pressure

$$\rho_{\text{average}} = \frac{M_{\odot}}{\frac{4}{3}\pi R_{\odot}^3} \approx 1410 \text{ kg m}^{-3}$$

← approximate, ρ does change with r .

$$\frac{dP}{dr} = -\rho(r) \left(\frac{GM}{R^2} \right) \quad \text{assume } \frac{dP}{dr} \approx \frac{P_{\text{surface}} - P_{\text{centre}}}{R} \text{ and } P_{\text{surface}} = 0$$

$$\text{so } \frac{dP}{dr} \approx -P_{\text{centre}}/R$$

Hence

$$P_{\text{centre}} \approx \frac{GM \rho_{\text{average}}}{R} \approx 2.7 \times 10^{14} \text{ N/m}^2 \quad \text{A lot!}$$

Adding in Ideal Gas Laws (Marcus)

$$P = nKT$$

n = No. Particles $\nwarrow$ Boltzmann's constant

μ = average mass of particles

$$\frac{1}{\mu} = \frac{m_H n}{\rho} \quad \text{for stars:}$$

m_H = Proton mass

$$\frac{1}{\mu} = 2x + \frac{3}{4}y + \frac{1}{2}z \approx 1.6$$

x = mass fraction of H

y is for Helium

z is metals.

The Sun's Central Temperature

rearrange the 2nd equation:

$$n = \frac{\rho}{\mu m_H} \quad \text{add in } P = nkT$$

$$T = \frac{P \mu m_H}{\rho k} \quad \text{all with respect to } \rho$$

using results before gives us $T_c \approx 1.2 \times 10^7 \text{ K}$.

The gas in the centre is ionised by neutral... it's plasma!

Cores have high enough temperatures to fuse light elements together. In each P-P chain reaction:

$$\begin{aligned} \text{Energy released} &= (4^1\text{H} - 4^4\text{He}) c^2 = (6.7 \times 10^{-27} - 6.6 \times 10^{-27}) c^2 \\ &= 4.3 \times 10^{-14} \text{ J} \quad \text{small but mighty!} \end{aligned}$$

So How Long Will The Sun Live (as type V)

≈ 10% of Sun's volume has fusion conditions.

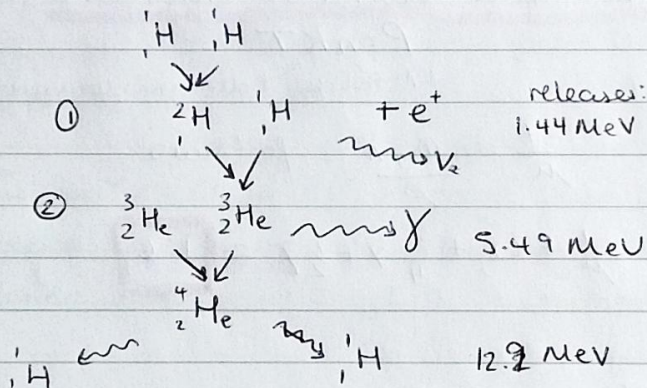
$$\text{Total Energy available} = \text{Energy per reaction} \times \frac{\text{total mass for fusion}}{\text{mass in each reaction}}$$

$$= 4.3 \times 10^{12} \times \frac{0.1 \text{ Mo}}{4 \text{ } ^1\text{H atoms}} = 1.07 \times 10^{44} \text{ J} \quad \uparrow \text{mighty}$$

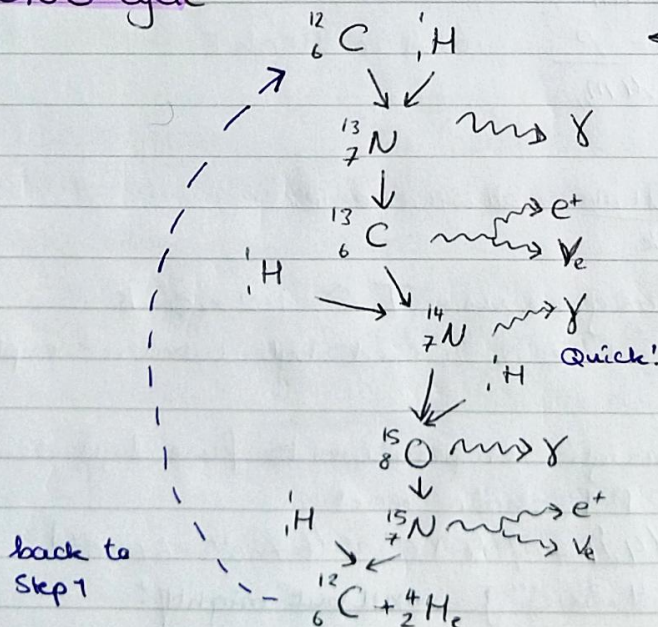
$$\text{lifetime} = \frac{\text{Total Energy}}{\text{Power output (L}_\odot)} = 3.3 \times 10^7 \text{ s or } 10^{10} \text{ years!}$$

Fusion

PP1 chain



CNO cycle



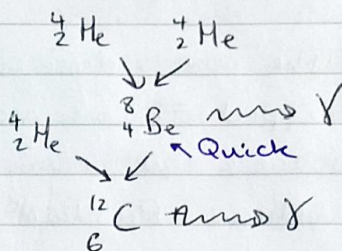
← Carbon isn't used

because it's a catalyst.

$$T > 1.6 \times 10^7 \text{ K}$$

It's for bigger stars (> 1.1 Mo), which have Carbon (population 1 stars).

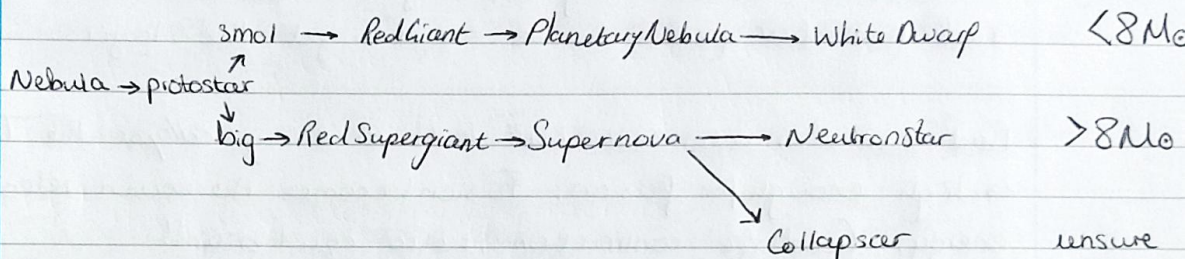
Triple α Big hot stars only! No babies!



Triple α is also similar for Ne, O, Mg, Fe.

Web

Life of Stars.



Birth

Stars are born in molecular clouds, like stellar nurseries.

Jeans Mass (mass for a cloud to collapse into a star)

for a cloud with mass M , radius R , N particles each with mass m and overall temperature T :

Potential Energy $U \approx -\text{constant} \frac{GM(Nm)}{R}$

Kinetic Energy $K \approx \frac{3}{2} NkT$

For collapse: $|U| > K$ so $\frac{GMNm}{R} > NkT$

Jeans Mass $\rightarrow M_J > \left(\frac{kT}{Gm} \right)^{3/2} R$

Molecular Cloud (H_2 Region) collapses, either from fluctuations or external agitation (shockwaves)

As it collapses, PE gets converted to KE + radiation,

KE is dissipated and radiation escapes, low temp & pressure, its a protostar.

Collapse continues until the core is dense and optically thick ($\tau \gg 1$)
Now radiation gets trapped $\Rightarrow$ core heats up, collapse halts, and eventually hydrostatic equilibrium is established.

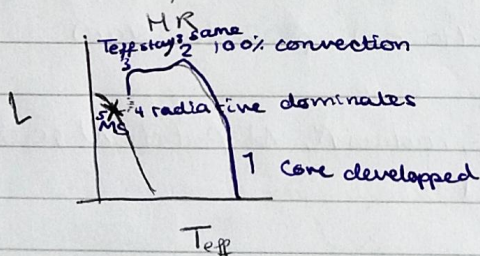
Now we have a Pre Main Sequence Star.

Temperature is low, and the star is basically 100% convective zone.

The Star contracts slowly, so the core heats up. The temperature at the surface doesn't change, so the luminosity drops and we start to see limb darkening (opacity $\downarrow$) The core initiates radiative transport, so we start to get the 3 layers.

Radiation dominates convection, but with enough collapse the Temp is high enough for fusion. Fusion becomes the main releaser of energy. Goodbye contraction, hello equilibrium!

The star is now a Zero Age MS star (ZAMS). This takes ≈ 20 million years.



Some stars have Protoplanetary Disks, which are cold gas & dust. Planets sweep out lanes in the disks

Mass - luminosity Relation

$$\frac{L}{L_0} = \left(\frac{M}{M_0} \right)^\alpha \quad \alpha = 2.3 \text{ for dimmer stars} \quad \& \quad \alpha = 4 \text{ more luminous stars at main}$$

below $0.43 M_0$ or $8 M_0$

Overall $\alpha \approx 3-3$ for everything else.

$$\left(\frac{t}{t_0} \right) = \left(\frac{M}{M_0} \right)^{-2.3}$$

time

so we can get an estimate for lifetime of a star during MS phase

In the Sun's evolution:

After 10 billion years, most of H is used up. T_{core} increases for He fusion and so the star brightens & expands.

But before that!...

Hydrogen is used up, core halts. Pressure drops and contractions start. The non-core H is pulled in and raised temperature. He fusion starts, but with a H shell around He core. P increases as $H \rightarrow He$ and adds to the core.

Next...

Too much He gets added, the core contracts. Energy generation goes turbo and the outer layers ('envelope that stores energy') expand.

The radius increases, but T_{eff} decreases, the Sun moves $\Rightarrow$ on HR.

The opacity has increased, energy carried by convection dominates, the radius keeps increasing and T_{eff} keeps decreasing. But because convection is so efficient, luminosity increases.

The star is moving to RG Branch. Contraction keeps occurring until $T_{\text{core}} \approx 100$ million K. Triple α starts. He fusion begins!

He
Triple α heats core quickly, because it increases temp, energy, and that speeds up rate of reaction. This thermal runaway is called Helium Flash, only a few minutes but uses 60-80% of Helium.

Core pressure increases, so it expands and cools, stopping He burning temporarily.

Outer layer & core contract again until T is enough for fusion again. He burnt in core, H burnt in shell. Nova subgiant

When core is Carbon, core contracts to burn outer He, expands again. This makes the star unstable & pulse, but it's nova giant on Asymptotic Giant Branch.

Thermal Pulses are cyclic. Core stops fusion, contracts, heats up enough, fusion again: expands.

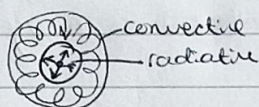
Energy gets rapidly transported to the surface via convection, sometimes so rapidly that it makes a superwind and removes all the outer layers. Planetary Nebula

lecture 7

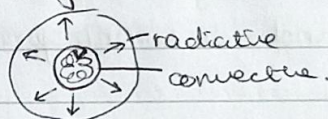
For Bigger Stars.

For very big stars, the main fusion is CNO ($T_{\text{core}} > 10 \text{ million K}$) where $4\text{H} \rightarrow \text{He}$ for only about 1 million years (O Type specifically)

Sunlike stars



Big stars



When H is gone, core contracts and H burns in a shell around He core. He core gets denser, contracts & heats up, H shell fuses quicker and outer layer expands. Temp cools.

Opacity increases, convection increases transport of energy to the surface. luminosity increases and moves to Red Giant Branch.

Core contracts until He fuses w/ triple α . NO Helium flash

He fusion

Core expands, luminosity drops. Slowly, Temp rises from He fusion. He in core is gone, Carbon C core contracts and heats up. The new H and He shell fuses faster and outer layer expands. All over.

C fusion

Small big stars:

unstable - heating &

cooling pulses stars

(Thermal pulses) will drive

a strong wind - ship material -

Planetary nebula. If bigger -

Supernova.

Bigger stars 5-8 $M_{\odot}$

Carbon core ignites $^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O} + \gamma$

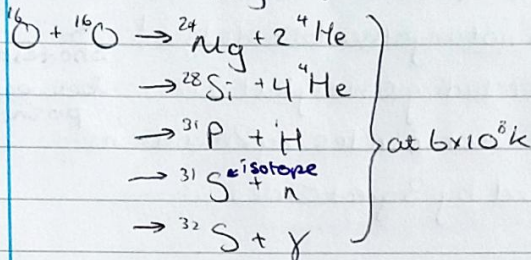
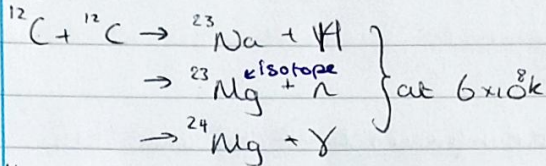
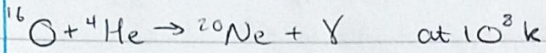
then oxygen maybe... Supernova

And even BIGGER stars $> 8 M_{\odot}$...

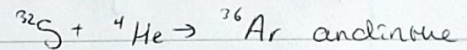
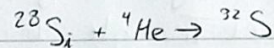
Big big stars > 8 Mo

large mass loss, luminosity mostly constant
mass loss $\propto (10^{-6} \text{ to } 10^{-7} \text{ Mo/yr})$

Burns H, He, C but also:



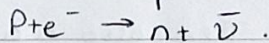
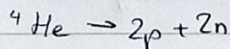
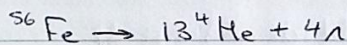
and other triplets



biggest stars ^{56}Fe

It can't go past ^{56}Fe because it is the biggest binding energy, anything bigger will need more energy to fuse, a waste of time

If the > 8 Mo star's core mass is bigger than Chandrasekhar limit (1.4 Mo) then electron degeneracy won't support the core. Now white dwarf, too fat to walk! neutron degeneracy occurs if $T > 6 \times 10^9$ by photodisintegration of ^{56}Fe and ^4He . It creates neutrons!

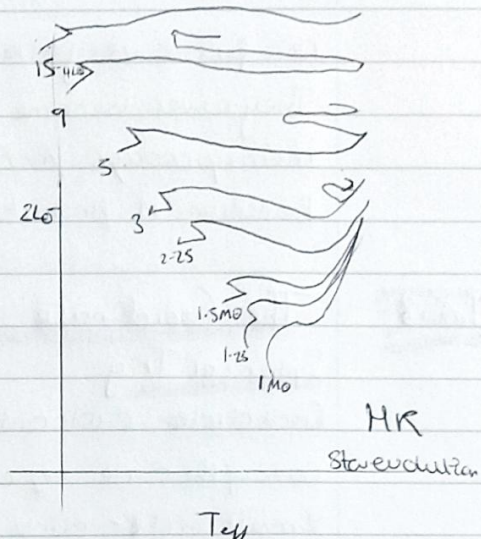


} requires 100 MeV.
core energy used quickly.

Neutron degeneracy - neutrons create pressure because it's so dense. the core can't collapse any more and creates shockwave. It trips apart like mad! Type II Supernova w/ neutron star

CHONK/STARS > 25 Mo

Neutron degeneracy can't help this big boy... black hole



Now for the very small stars $< 0.08 M_{\odot}$

They never become MS stars. Gravitational collapse doesn't heat them up enough for PP1. If they're $> 14 M_{\text{Jup}}$ or $> 14 M_{\text{J}}$ can fuse Deuterium ^2H for short time. These stars cool off and are Brown Dwarves.

Lecture 8

The Coordinate System

Spherical Trig

Great circle - circle on the sphere (circumference) where the centre is the centre of the circle eg equator and not any other latitude line!

Small circle - circle on sphere not including centre eg latitude.

Shortest distance between 2 points

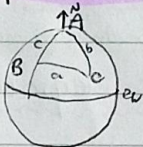
Spherical angle - angle between 2 Great circles eg 2 meridians.

Spherical Triangle - triangle bound by 3 great circles.

1 nautical mile = 1 arc minute $1/60^\circ$ along a great circle

eg 0°N to 90°N North $\rightarrow$ Equator $90^\circ = 5400' = 5400$ nautical miles. $1 \text{ Nm} = 1.852 \text{ km}$ so from 0°N to $90^\circ \text{N} = 101000.8 \text{ km}$

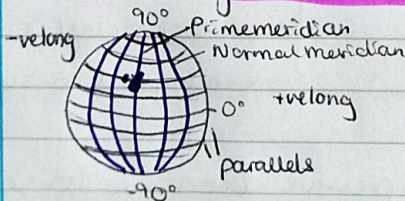
Spherical law of Cosines:



$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Similar to cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Coordinate system 1: latitude & longitude



There is also a Paris meridian, 'cause they were mad over Bonaparte's exile.

How to write angles.

Degree Decimal Oadby Scopes 52.63°N

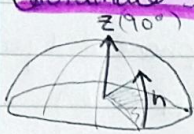
Sexagesimal notation - integer degree, arc minutes, arc seconds

eg Oadby = $01^\circ 07' 25.6'' \text{W}$, $52^\circ 37' 22.4'' \text{N}$

longitude in terms of time - $1 \text{ hour} = 15^\circ$ so $1 \text{ degree} = 4 \text{ minutes}$

- Oadby Scopes is $-0\text{h} 4\text{m} 29.7\text{s}$. take decimal, $\div 15$, then convert back to sexagesimal.

Coordinate system 2: Altazimuth



altitude h = angular distance of object above horizon.

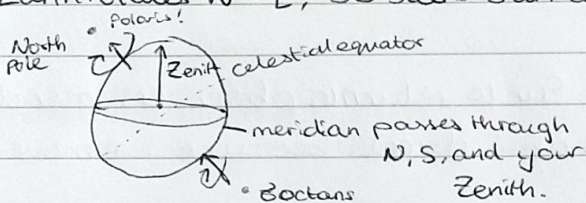
Zenith distance = $90 - h$

Azimuth A = angular bearing of object East.

Altazimuth always changes with Earth's rotation!

The celestial Sphere

Imaginary infinite sphere w/ you at centre
Earth rotates $W \rightarrow E$, so stars sun etc move $E \rightarrow W$.



North pole is at angle ϕ = latitude

Circumpolar stars never set (within ϕ at latitude ϕ)

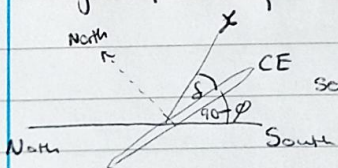
When a star crosses the meridian, it transits it and reaches maximum altitude.

Stars within $-\phi$ (for North observer) will never be seen above horizon.

Coordinate system 3: Declination (latitude)

DEC 0° equator, -90° N/S pole. measured from equator.

highest point of star is $90^\circ - \phi$. Declination = $(90^\circ - \phi) + \delta$



so angle of star is $(90 - \phi) + \delta$ due South

If the star is due North

we use $180 - (90 - \phi) + \delta$ due North.



Right Ascension (longitude)

time. 0h 0m 00s = First Point of Aries (Y) occurs at vernal (spring)

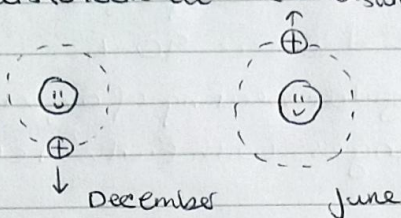
Equinox when Sun's position moves from South hemi to North.

Lecture 9

Motion of The Sun

Apparent position of the Sun moves around the sphere in 1 year.

Best stars to look at $\alpha = \alpha_{\text{sun}} + 12\text{hr}$ (directly opposite)



The Zodiac are the constellations where the Sun passes through.

Sidereal Day

Time taken for stars to return to exact place in sky (360°)

23h 56m 4.09s

Solar Day

Synodic day - time for Sun to return to observer's meridian.

24 hrs. (a bit more than Sidereal because we move in an orbit)

The Ecliptic

and most planets they formed from the same disk!

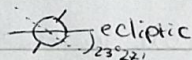
The path the Sun follows (plane of Earth's orbit) Earth's spin axis = $23^\circ 27'$ called the obliquity.

The ecliptic is at 0 Dec (equator) when:

- ° Vernal Equinox 21 or 22 March (First point of Aries)
- ° Autumnal Equinox 21 or 22 September in Libra.

The ecliptic is max at:

June 21/22 or Dec 21/22 Solstice at $+23^\circ 27'$ and $-23^\circ 27'$



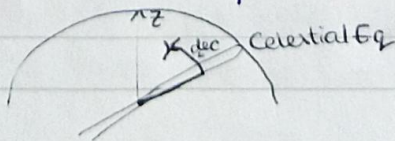
Lecture 10

Hour Angle HA Good to know which stars should be good to observe

star's position relative to observer's meridian. It increases Westwards so with time. -12h to +12h. Stars with -ve HA are yet to rise.

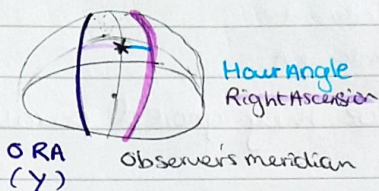
= RA of star - Observer's Meridian in hours

Tip Good stars are within 3hrs of meridian



Sidereal Time

HA from ORA (γ) as a measure of Sidereal Time. The hourangle from γ is local Sidereal Time LST.



$$LST = HA + RA$$

when $0 = HA$ then $RA = LST$ when it is highest in sky. Stars w/ this are best to see.

Leicester LST = 03h22m tonight (13/11) midnight.

Lat = $52^\circ 38' N$

Long = $1^\circ 8' W$

Universal Time

UT - Greenwich Mean Time at 0° longitude.

textwell

Geometric Parallax

angular displacement relative to background objects

π_g used.

$$d = \frac{\Delta}{\pi_g} \quad \begin{array}{l} \text{displacement} \\ \text{distance} \end{array} \quad \begin{array}{l} \text{parallax} \end{array}$$



$$\frac{\Delta}{2} = d \tan \frac{\pi_g}{2}$$

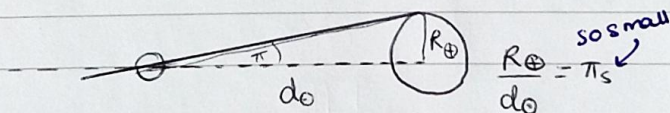
$$d = \frac{\Delta}{2 \tan \frac{\pi_g}{2}}$$

$$\text{approx } \tan \frac{\pi_g}{2} \approx \frac{\pi_g}{2}$$

$$d = \frac{\Delta}{\pi_g}$$

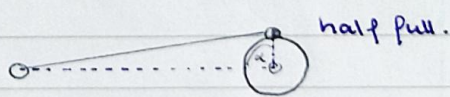
Solar Parallax

Angle subtended at Earth's radius at the distance of the Sun:



This can't be done, because π_s is so small and there are no background stars during the day. To find $d_{\odot}$, Aristarchus measured:

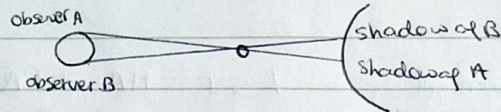
$$\text{dist } \frac{d_{E \text{ to moon}}}{d_{E \text{ to sun}}} = \cos \alpha$$



Moon's terminator - sharp half line between light & dark side when half full. He got it very wrong!

Another Method: Venus' Transit

As Venus transits it produces a shadow. 2 far away observers will see the shadow displaced due to parallax.



3 scientists tried, but failed coz they ignored parallax.

Trigonometric Parallax

parallax = $\frac{1}{2}$ angular change at a distance of 1 parsec.

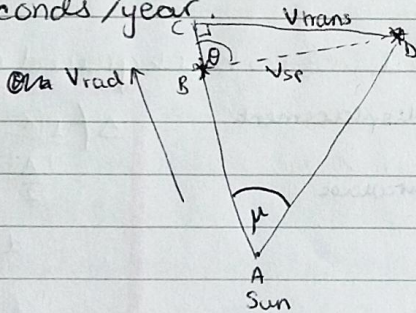
1 AU $\Rightarrow$ 1 arcsecond

$$d = \frac{1}{\pi''}$$

Parsec $\leftarrow$ arcseconds.

Proper Motion

Stars drift in position across sky. proper motion μ is measured in arcseconds/year.



Star moves from B $\rightarrow$ D in 1 year.

$$\sin \mu = \frac{V_{trans}}{d}$$

if μ is small $\mu = \frac{V_{trans}}{d}$ AU per year.

if $\pi = \frac{1}{d}$ then $V_{trans} = 4.74 \frac{\mu''}{\pi''}$