Unit 2: Circuits

Lecture 1

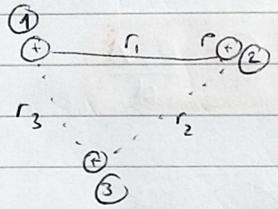
Electrostatic potential Energy

(+) r .Point p   so $V = \frac{kq}{r}$

if we brought another charge (+) to p from ∞, we need to do work: $W = qV$

$$\boxed{W = \frac{kq_1 q_2}{r}}$$

If we want a 3rd charge:



more repulsion = more work

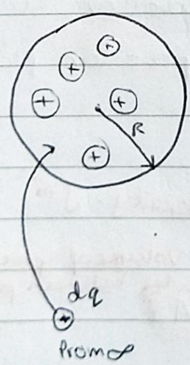
$$W = \frac{kq_1q_3}{r_3} + \frac{kq_2q_3}{r_2}$$

So the total system potential energy:

$$\boxed{U = \frac{kq_1q_2}{r_1} + \frac{kq_1q_3}{r_3} + \frac{kq_2q_3}{r_3}}$$

Simplify! $\boxed{U = \frac{1}{2}(q_1V_1 + q_2V_2 + q_3V_3)}$

means POV of 3rd particle, so find V for other 2.



If you load a conductor with charge Q, it acquires a potential V.

$$V = \frac{kQ}{R}$$

The work done dU to bring little charge to the conductor is $dU = Vdq = \frac{kQ}{R}dq$

so $U = \frac{k}{R}\int_0^Q Q\,dq = \frac{kQ^2}{2R} = \frac{1}{2}QV$

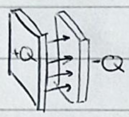
Capacitance

We know: $\boxed{C = \frac{Q}{V}}$

For spherical conductors:

if $V = \frac{kQ}{R}$ $\quad C = \frac{QR}{kQ} = 4\pi\varepsilon_0 R \xleftarrow{\text{or}} \left(\frac{R}{k}\right)$

Parallel Plates (classic capacitor)

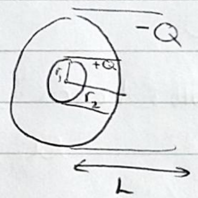


$E = \frac{Q}{\varepsilon_0 A}$ outside the surface of a conductor

$$\boxed{V = Ed = \frac{Qd}{\varepsilon_0 A} \quad \text{so} \quad C = \frac{\varepsilon_0 A}{d}}$$

Cylindrical Capacitor

Used in coax cables



$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_2}{r_1}\right)}$$

It takes energy to charge a capacitor. Capacitors store energy as an electric field between the 2 capacitor plates

$$U = \tfrac{1}{2}QV$$

& $\quad C = Q/V$

so $\quad U = \tfrac{1}{2}CV^2.$

$\downarrow \quad \downarrow V = Ed$

$$U = \frac{1}{2}\frac{\varepsilon A}{d}(Ed)^2 = \boxed{\underbrace{\tfrac{1}{2}\varepsilon_0 E^2}_{\text{energy density} \cdot Jm^{-3}} Ad}$$

volume of space between plates m^3

lecture 2

Example: Cameras!

Energy of 1 flash = 300 watts × 3 ms = 0.9 J for 100 v camera

so $U = \tfrac{1}{2}CV^2 \therefore C = \frac{2U}{V^2}$

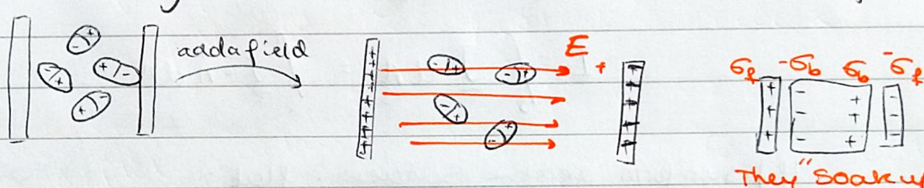
hence $C = \frac{2 \times 0.9}{100^2} \approx 2 \times 10^{-4}$ F that's a lot!

How to increase capacitance:

- Increase Area (increases charge) by rolling it up.
- Smaller distance between plates (decrease V)
  - Increases E, so be careful it can't go over 3kV and it discharges.
- Add a dialectric
- Use capacitors in parallel.

Dialectrics

non conducting material that can be polarised by an electric field.



They "soak up" or diminish some electric field, we can decrease V more! No breakdown!

$$E = \frac{E_0}{\kappa} \leftarrow \text{kappa} \qquad \therefore V = \frac{V_0}{\kappa}$$

$$C = \kappa C_0$$

increased capacitance — scale factor — capacitance it would have sans dialectric

Aire: $E_{max} = 3kV/mm$

Plexiglass: $E_{max} = 40$

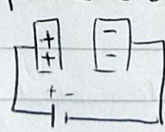
$\kappa = 3.4$.

Batteries.

Store energy through chemical reactions,

produce a PD between 2 terminals called terminal voltage

Charges build up at plates to diminish PD



In circuits:

in serial

$$C_{eq} = \frac{Q}{V_1 + V_2}$$

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

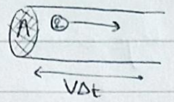
in Parallel

$$Q_{total} = Q_1 + Q_2$$
$$= (C_1 + C_2)V$$

$$C_{equivalent} = C_1 + C_2$$

Current:

rate of flow of charge: $I = \frac{\Delta Q}{\Delta t}$ Ampère = $C\,s^{-1}$



Charge density = n

so total charge = qn

total volume = $A v \Delta t$

total charge in volume $\Delta Q = qnAv\Delta t$ Malcolm

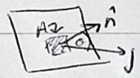
Hence $I = qnvA$ where in a circuit $I = nAve$

Current Density $\vec{J} = n\vec{v}q$

$$I = \int_S \vec{J}\, d\vec{A} = \int_S \vec{J} \cdot \hat{n}\, dA$$

If J is uniform and the surface is flat:

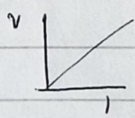
$$I = J A \cos\theta$$



Ohm's Law

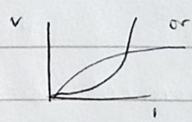
$\Delta V = E \Delta L$ and $R = V / I$

$V = E\,d$



Ohmic

used for resistors



or something else

~~Vario~~ non-Ohmic

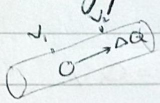
diodes, LED, lamps.

Resistivity $\boxed{R = \rho \frac{L}{A}}$ (ρ fundamental property)

$\frac{1}{\rho}$ = Conductivity σ units Ωm.

Energy in Electric Circuits.

Lecture 3



$$\Delta U = \Delta Q (V_2 - V_1) = \Delta Q V$$

over time Δt:

$$P = \frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} V = IV$$

$\Delta Q = I \Delta t$

Power lost due to Joule heating in a resistive conductor

$$\boxed{P = I^2 R = V^2/R}$$ ← add $V = IR$

Joule heating is like electric friction. Friction imposed by atoms in the conductor on the moving charges.

A battery does work on charges by raising the charges through the potential between its terminals.

not a force!

Work done per charge = electromotive force $\mathcal{E}$ (in V)

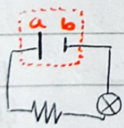
$$P = \frac{\Delta Q \mathcal{E}}{\Delta t} = I\mathcal{E}$$

real batteries aren't 100% efficient. it uses some of the circuits energy & it has its own resistance, so:

$R = r$

$$V_a - V_b = \mathcal{E} - \boxed{Ir}$$ internal resistance

Terminal (battery) voltage

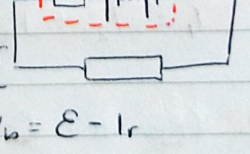


As we increase current, Voltage drops in circuit

In a Simple circuit:

$V = V_a - V_b = \mathcal{E} - Ir$ {

line shows mini resistor is just internal resistance.



$V = IR$ {

so $IR = V_a - V_b = \mathcal{E} - Ir$

$\mathcal{E} = I(R + r)$ in series, so $R_{tot} = R_1 + R_2$

similar to $\frac{V}{R}$ - - - → $$\boxed{I = \frac{\mathcal{E}}{R + r}}$$

Energy Stored in a battery:

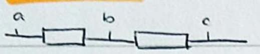
$$\boxed{E_{stored} = Q\,\mathcal{E}}$$

charge delivered over its LIFETIME

Similar to $E = QV$

Behaviour in resistors:

In Series



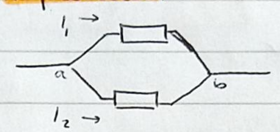
$V_{total\ ac} = V_{ab} + V_{bc}$

$V = IR$

$V = IR_1 + IR_2$

$$\boxed{R = R_1 + R_2}$$

In parallel



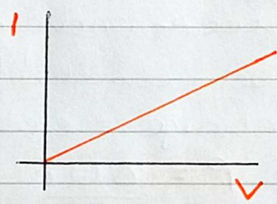
$I = I_1 + I_2$

$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Ohmic components

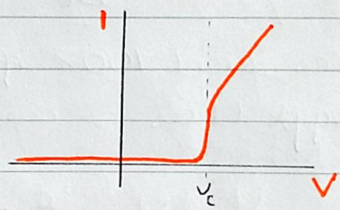
where $V = IR$



Diodes

$R \sim 0$ above V_c

$R \sim \infty$ below V_c

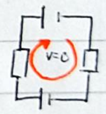


Diodes are good to make AC!

Kirchoff's Laws

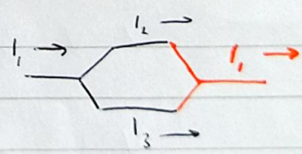
1. Sum of all Voltages around a closed loop $= 0$

$$\oint \underline{E} \cdot d\underline{r} = 0$$



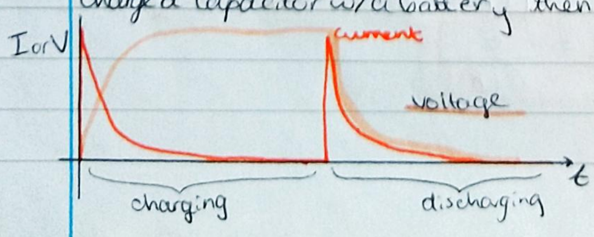
2. Total Current in $=$ total current out

$$I_1 = I_2 + I_3$$

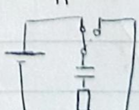


RC Circuits (Resistor Capacitor)

Charge a Capacitor w/ a battery then switch it off to discharge:



For



discharging current can go -ve sometimes, depends on direction.

Charging & Discharging

Big Voltage between plates.
Big Voltage = Big Current
so large flow of charge.
$V = \frac{Q}{C} = \frac{dQ}{dt} R$
-1

⇒ Charge builds up, it equalises. Smaller charge = Small Voltage = Small Force = Small current

⇒ **Discharging**
Exponential Decrease
in current & voltage when discharging

Discharging Equations

$$\frac{Q}{C} = -\frac{dQ}{dt} R$$

$$\frac{dQ}{dt} = -\frac{Q}{RC} \Rightarrow \frac{dQ}{Q} = -\frac{1}{RC}\, dt$$

$$\int_{Q_0}^{Q'} \frac{dQ}{Q} = -\frac{1}{RC} \int_0^{t'} dt$$

$$\ln Q'/Q_0 = -\frac{t'}{RC}$$

$$Q = Q_0 e^{-t/RC}$$

$RC = \tau$ sometimes

Hence:

$$I = I_0 e^{-t/RC}$$

0 voltage between plates. Large current since no force opposing emf.

⇒ charge builds up Voltage increases, opposes emf, so I decreases.

⇒ **Charging**
Opposite exponential decrease.

Charging Equations:

$$\mathcal{E} - \frac{dQ}{dt} R - \frac{Q}{C} = 0$$

$$Q = C\mathcal{E}(1 - e^{-t/RC})$$

Hence:

$$I = I_0 e^{-t/RC}$$