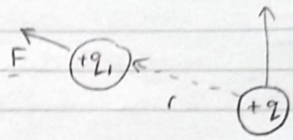
Suzie F646

Lecture 1

Electromagnetic Fields

Electrostatics - recap from year 1.

$\underline{E} = \frac{F}{q}$ $\quad \underline{F} = \frac{q_1 q}{4\pi\varepsilon_0 r^2}\,\hat{\underline{r}}$



for more $q_2\, q_3$ etc add all the effects if you want force due to centre q.

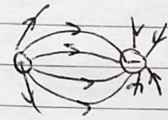
$F = -\sum \frac{q q_i}{4\pi\varepsilon_0 r^2}\,\hat{r}$ principle of summation hence => $\underline{F} = q\underline{E}$.

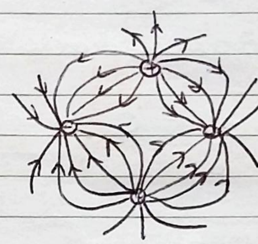
Fieldlines

- lines of force
- +ve to -ve
- Can't cross

Strongest places have closer lines.

Dipole

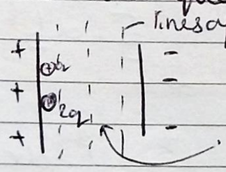




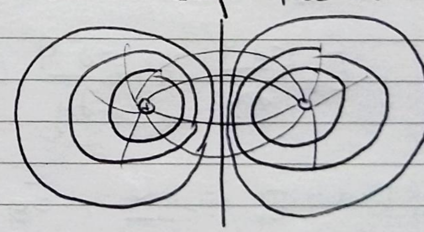
quadripole.

Potential

Scalar quantity. Same as gravity, but with repulsion.



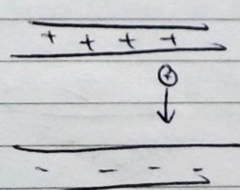
Lines of potential are contours of field



potential φ:

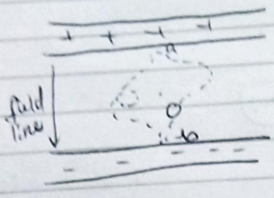
$$\varphi_E = \frac{-1}{q}\int_{\infty}^{r} F\,dr'$$

and $\underline{E} = -\nabla\varphi$ $\quad \varphi$ can be used as potential too.



field does the work
potential decreases

Doing work in an electric field (electrostatic only)



the work done by the electric force is the same for any path from a to b:

$W_{a \to b} = -\Delta U$ so just the use displacement, all the squiggles don't matter.

work done = change in potential

Same for gravity.

$$\boxed{W_{AB} = \int_A^B \underline{F} \cdot d\underline{l} = -q\int_A^B \underline{E} \cdot d\underline{l} = U_B - U_A}$$

only ~~orthogonal~~ parallel displacements matter.

Force & electric field are conservative + path independant. just like being born and dying!

If we integrate a field around a closed loop:

$$\boxed{\oint \underline{E} \cdot d\underline{l} = 0}$$ electrostatic only.

$\varphi = U/q$ so like a gradient of field.

we have:

$\underline{E} = -\nabla \varphi$

hence:

$E_x = \frac{-\partial \varphi}{\partial x}, E_y = \frac{-\partial \varphi}{\partial y}, E_z = \frac{-\partial \varphi}{\partial z}$

or just use V

and $E_r = -\frac{\partial \varphi}{\partial r}, E_\theta = -\frac{1}{r}\frac{\partial \varphi}{\partial \theta}$

polar

The zero of a potential is defined as the value at infinity:

$$\boxed{V = -\int_\infty^r \underline{E} \cdot d\underline{l}}$$

$$= \int_r^\infty \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} \cdot d\underline{r}$$

$$V = \frac{q}{4\pi\varepsilon_0}\int_r^\infty \frac{1}{r^2} dr \boxed{= \frac{q}{4\pi\varepsilon_0 r}}$$

Since $E = -\nabla\varphi$

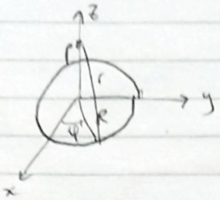
using polar potential is independant of θ so $E_\theta = 0$

from here

now $\underline{E} = E_r \hat{r} = \frac{-d}{dr}\left(\frac{q}{4\pi\varepsilon_0 r}\right)\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$

back to way we started.

Ring of charge



$$dq = \lambda\, dl = \lambda R\, d\varphi'$$

$$\underset{\text{potential}}{dV} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} \quad \text{and } r = \sqrt{R^2+z^2}$$

subs

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\, d\varphi'}{\sqrt{R^2+z^2}}$$

So now potential from entire ring:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{\sqrt{R^2+z^2}}\int_{\varphi'=0}^{2\pi} d\varphi'$$

$$\boxed{= \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}} \qquad \boxed{Q = 2\pi R\lambda}$$

if $z >>> R$ $\quad \sqrt{R^2+z^2} \Rightarrow z$. Looks like potential from point charge.

If we want E_z $= \dfrac{-Q}{4\pi\varepsilon_0}\dfrac{\partial}{\partial z}\left(\dfrac{1}{\sqrt{R^2+z^2}}\right)$

$$E_z = \frac{Q}{4\pi\varepsilon_0}\frac{z}{(R^2+z^2)^{3/2}} \quad \mathrm{V\,m^{-1}}$$

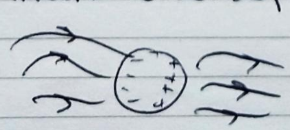
no electric field exists within a conductor.
Free electrons move to cancel out the field.

Lecture 2

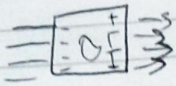
Insulators & Conductors

Conductors - free electrons - sit evenly around charged surface. no electric field inside - always normal to surface. But there's a field outside.

Surface is an equipotential
In an electric field, electrons move.
polarisation.



If there's a hole



no field in conductor - no field in hole.

Flux

$\phi_E = \underline{E} \cdot \underline{A}$ area that's perpendicular ($A\cos\theta$)

$$\boxed{\phi = \int_S \underline{E} \cdot d\underline{S} = \iint_S \underline{E} \cdot \hat{n}\, dy\, dz}$$

Jacobians

cylindrical volume $r\,dr\,d\varphi\,dz$ Area $r\,d\varphi\,dz$ + $r\,dr\,d\varphi$

curved — two ends of cylinder.

Spherical $r^2\sin\theta\,dr\,d\theta\,d\varphi$ Area: $r^2\sin\theta\,d\theta\,d\varphi$

$\theta \to \pi$ $\varphi \to 2\pi$

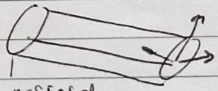
Gauss' Law

flux through closed surface = sum of charges within the surface.

$$\boxed{\phi = \iint \underline{E} \cdot d\underline{S} = \oint_S \underline{E} \cdot d\underline{S} = \frac{Q_{enclosed}}{\varepsilon_0}}$$

eg a wire, use cylinder as Gaussian surface: $Q_{inside} = \lambda L$

$$\phi = \int_{z=0}^{L}\int_{0}^{2\pi} E(r)\, r\, d\varphi\, dz \qquad \phi = 2\pi r L\, E(r)$$



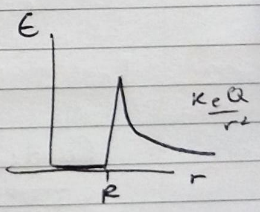
no field at ends.

$$\text{so } \frac{Q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0} = \phi$$

$$E(r) = \frac{Q}{2\pi r L \varepsilon_0}$$

eg a sphere.

$$\phi = \iint_{\theta\ \varphi} R^2 \sin\theta\, d\theta\, d\varphi$$

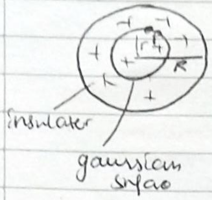


$$\phi = E(r)\, 4\pi R^2 = Q/\varepsilon_0$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0 R^2}$$

Insulators

charge is uniformly destributed.

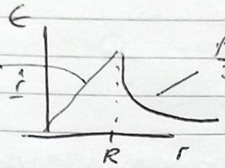


Charge density inside sphere $= \rho$

$$E = \frac{r\rho}{3\varepsilon_0}\hat{r} \qquad r \leq R.$$

$$Q_{enc} = \frac{4}{3}\pi r^3 \rho$$

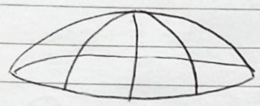
for gaussian surface only, not outer insulator.



$E = \frac{\rho r}{3\varepsilon_0}\hat{r}$ $\qquad \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}$

Lecture 3

Example flux through hemisphere with base



$\uparrow E$ E isn't normal to surface anymore.

$$\text{Flux} = \int_{\varphi 0}^{2\pi}\int_{\theta 0}^{\pi/2} R^2 \sin\theta \, d\theta \, d\varphi.$$

field at certain point

$\underline{E} \circ \hat{\underline{n}} = a\cos\theta$

So $\phi = \iint a\cos\theta \, R^2 \sin\theta \, d\varphi \, d\theta$ } cure dbl-

do θ part with substitution

$$\phi = aR^2 [\varphi]_0^{2\pi} [\tfrac{1}{2}\sin^2\theta]_0^{\pi/2}$$

$= 1/2$

$$= \pi a R^2$$

$E \cdot \hat{n} = a$ completely through } base

$$\phi = \int_0^R \int_0^{2\pi} a r \, dr \, d\theta = a\pi R^2$$

total $\phi = 2\pi a R^2$.

Dielectrics

Recap: Divergence Theorem

$$\oiint \underline{F} \cdot d\underline{s} = \iiint \nabla \cdot \underline{F} \, dV$$

Gauss' Law:

$$\oint_S \underline{E} \cdot d\underline{s} = \frac{Q}{\varepsilon_0} = \int \nabla \cdot E \, dV$$

$$\boxed{\oint_S \underline{E} \cdot d\underline{s} = \frac{1}{\varepsilon_0} \int_V \rho \, dV}$$

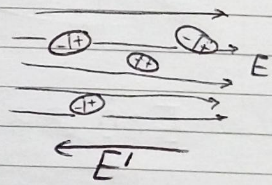
$Q_{enc} = \int_V \rho \, dV.$

$$\boxed{\nabla \cdot \underline{E} = \frac{\rho}{\varepsilon_0}}$$

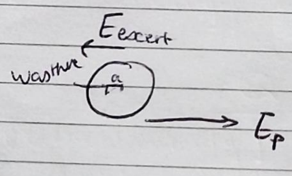
¡¡must rememeber!!

Dielectrics have fewer free electrons. React by becoming polarised when applying a field.

Polar dielectric - molecules rotate to align with field generates opposing field E'



Permanent dipoles. due to electronegativity in molecules.
Can be done with stretching also. (pull e^- one way).



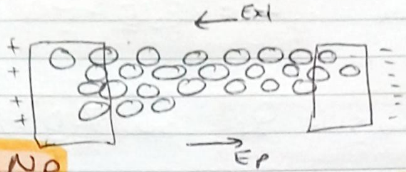
dipole moment $\boxed{\underline{p} = Ze\underline{a}}$ in Cm.

$\underline{a}$ = displacement.

$$\boxed{\underline{p} = \alpha \underline{E}_{ext}}$$

α = Polariseability - property of element.

If we move a solid structure polarisation happens at ends.



$\sigma_p = -N_p$

$\sigma_p = +N\underline{p}$

dipole mo

number of atoms

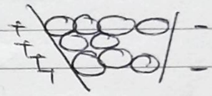
surface charge density

$\sigma_p \delta S = N\underline{p} \cdot d\underline{S}$

$$P = Np$$

dipole moment per unit volume / polarisation in Cm^{-2}

one end might have more charge



$$Q = \underline{P}\, \delta S \cos\theta = \int_S \underline{P} \cdot d\underline{s}$$

using divergence theorem

charge per unit volume $\rho_p = -\nabla \cdot \underline{P}$

Dielectric Susceptibility

χ_e - electric susceptibility

$$\underline{P} = \varepsilon_0 \chi_e \underline{E_{ext}}$$

relative permettivity

$$\varepsilon_r = (1 + \chi_e)$$

$$\underline{P} = (\varepsilon_r - 1)\varepsilon_0 \underline{E}$$

applied field.

Gauss' Law for dielectrics

free electron charge

$$\oint E \cdot d\underline{s} = \frac{Q_f}{\varepsilon_0} + \frac{Q_p}{\varepsilon_0}$$

polarisation "

$$\oint_S (\varepsilon_0 \underline{E} + \underline{P}) \cdot d\underline{S} = Q_f \quad \& \quad \nabla \cdot (\varepsilon_0 \underline{E} + \underline{P}) = \rho_f.$$

electric displacement $\boxed{\underset{(Cm^{-2})}{\underline{D}} = (\varepsilon_0 \underline{E} + \underline{P})}$

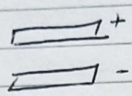
So for Gauss' Law:

$\boxed{\oint_S \underline{D} \cdot d\underline{S} = Q_f}$ so $\boxed{\nabla \cdot \underline{D} = \rho_f}$

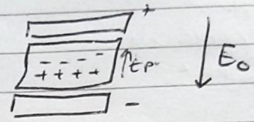
$\underline{P} = \varepsilon_0 \chi_e \underline{E}$

$\underline{D} = \varepsilon_0 (1 + \chi_e) \underline{E} = \varepsilon_0 \varepsilon_r \underline{E}$

How dielectrics work



now



total $\underline{E} = \underline{E}_0 + \underline{E}_P$

So now E is a little lower. so $\underline{P} = \varepsilon_0 \chi_e \underline{E}_0 = (\varepsilon_r - 1) E_0$

reduce $\underline{E}$, reduce V hence from $Q = CU$, C increases.

$C = \varepsilon_r C_0$.

- They stop plates from coming into contact
- Reduce possibity of shorting by sparking (dielectric breakdown)

Lecture 4

Magnetism

Flux, = field through surface $\phi_E = \int_S \underline{E} \cdot d\underline{S}$ and $\oint_S \underline{E} \cdot d\underline{S} = \frac{Q}{\varepsilon_0}$

closed, Gauss' Law

Current density J : $\boxed{I = \int_S \underline{J} \cdot d\underline{S} \qquad \underline{J} = nq\underline{v}}$

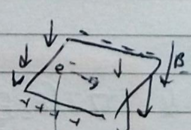
$d\underline{F} = I(d\underline{l} \times \underline{B})$ unit of magnetic field = force experienced by current element in magnetic field.

① $\underline{F} = q(\underline{V} \times \underline{B})$

② $\underline{F} = q\underline{E}$

So together ① and ②:

$\boxed{\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})}$

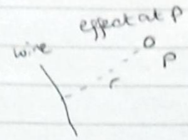


Hall Effect charges feel F_B and also F_E combine:

$-e\underline{E} =$

flowing e start to move to charge the plate

Biot Savart $dB = \frac{\mu_0 I \, d\underline{S} \times r}{4\pi r^2}$



Amperes law $\oint \underline{B} \cdot d\underline{l} = \mu_0 I$

$= \int_S \underline{\nabla} \times \underline{B} \cdot d\underline{S} = \mu_0 \int \underline{J} \cdot d\underline{S}$

$\Rightarrow \underline{\nabla} \times \underline{B} = \mu_0 \underline{J}$

Lecture 5

Gauss' Law

$\oint_S \underline{B} \cdot d\underline{S} = 0$ No magnetic monopoles!

differential form (use this mostly) (divergence theorem)

$\int_V \nabla \cdot \underline{B} \, dV = 0.$

$\nabla \cdot \underline{B} = 0$

Flux $\Phi = \int_S \underline{B} \cdot d\underline{S}$ no net flux for closed surfaces, but open ones can!

Faraday's Law

If a surface changes so Φ_B changes, a current is induced to reverse the change (remember direction).

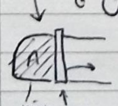
The EMF for this current:

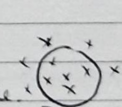
$$\mathcal{E} = -\frac{\partial \Phi_B}{\partial t}$$ in Volts.

How to change flux:

$\nabla \times \underline{B} = \mu_0 \underline{J}$ So if $\underline{B}\downarrow$, $\underline{J}\uparrow$ to increase B again

- Change field strength
- Increase wire / current loop size. ← more / less flux flowing inside.
- change angle of wire / loop ← change normal component





it I goes ← its B field (RH Grip) increases B inside circle

$$\oint_C \underline{E} \cdot d\underline{l} = -\frac{\partial}{\partial t} \int_S \underline{B} \cdot d\underline{S}$$

Differential form: Stoke's Theorem

$$\int_S \nabla \times \underline{E} \cdot d\underline{s} = -\frac{\partial}{\partial t}\int_S \underline{B} \cdot d\underline{S}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

if youre using LHR or RHGrip with Lorentz force (ie with electrons and velocity), ~~then~~ it already accounts for e^- direction. CHECK.

Eddy Current

Current changes in metal pipe to reduce magnetic field of magnet → Lenz Law. Eddy currents in the pipe.

Brakes make Eddy currents in metal plate.
Metal detectors make eddy currents in treasure.
Also used in induction hob. (thats why you need magnetic Saucepan!!!)

Sun's wind Squashes Mercury's magnetosphere so it makes eddy currents to combat. Only Mercury does this because of ginormous core!

Lecture 6

Lecture 6

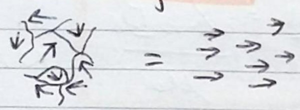
Materials

Electrons have angular momentum / spin. Either spin up or down. So Pauli exclusion principle => pair of up down electrons. Magnetic fields cancel out here.
Unpaired all up/down spins makes a magnetic field.

Diamagnetic - eg Water. Acquires B field under influence of external field. Opposite to external field.

Paramagnetic - Aluminium. permanent dipole moment. Apply a field, atoms line up IN direction of external field

Ferromagnetic - Iron. Align far stronger than paramagnetic. Made of small patches called domains. can be 1000 × external field.



Why do ferromagnets align so well?

Long range ordering. Patches line up inside, then patches align w/ each other. Boundaries disappear.

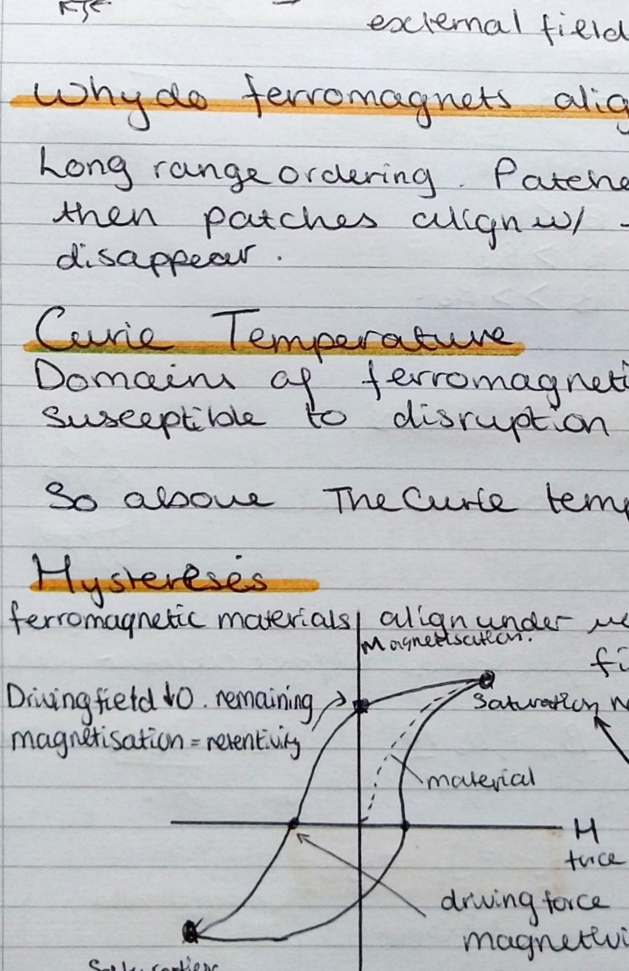
Curie Temperature

Domains of ferromagnetic materials are susceptible to disruption through heating.

So above the Curie temp, ferro => para.

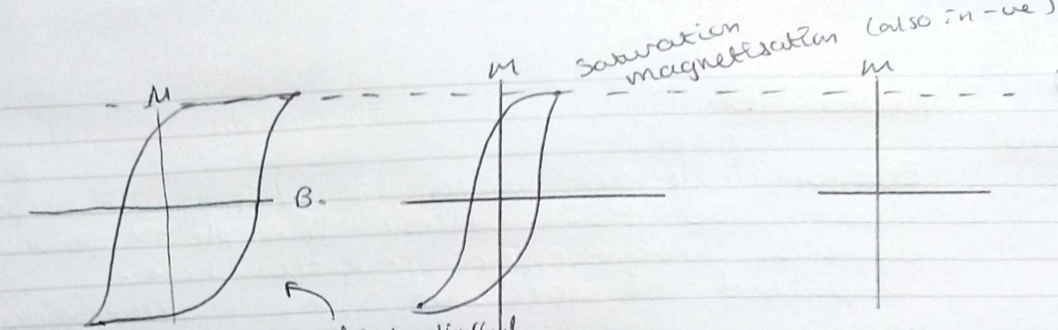
Hystereses

ferromagnetic materials | align under weak magnetic field. When field is ~~aligned~~ removed, domains remain aligned, still magnetic. Called hysteresis.



Saturation. All domains aligned & eliminated.

driving force must be reversed to bring magnetivity to zero = coercivity.



retains large amount of saturation field.
Good for permanent magnets & recording devices w/memory

narrow => Small dissipated energy in repeatedly reversing magnetisation
Good for transformer & motor cores.
to minimise energy dissipation with alternate fields associated with AC.

Magnetisation.

largescale magnetisation => sum of small contributions

$$\boxed{\underline{M} = N \langle \underline{m} \rangle}$$ in Am^{-1}.

$$\boxed{\underline{M} = \frac{\chi_B \underline{B}_0}{\mu_0}}$$ NA^{-2}.

total magnetic field:

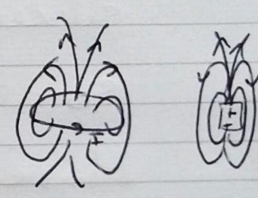
$$\underline{B} = \underbrace{B_0}_{\text{external}} + \underbrace{B_m}_{\text{induced.}}$$

Example: MRI

diamagnetic	$B < B_0$
paramagnetic	$B > B_0$
ferromagnetic	$B >> B_0$.

Currents in Magnetic fields.

Magnetic dipoles look like little current loops.

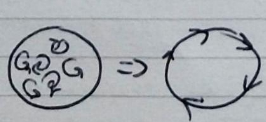


In a solid, internal current loops cancel => net current only on surface. Magnetisation surface current density gives B_m inside:

$$B_m = \mu_0 \underline{M} \qquad \underline{M} = \frac{\chi_B B_0}{\mu_0} \qquad \nabla \times \underline{M} = \underline{J}_M$$

Surface current density of magnetisation, J_M.

Similar to $\nabla \times \underline{B} = \mu_0 \underline{J}$



$$\boxed{\underline{J} = \underbrace{\underline{J}_f}_{\text{free charges}} + \underline{J}_M} \leftarrow \text{magnetisation}$$

Magnetic Intensity:

$$\boxed{\begin{array}{c} \underline{H} = \dfrac{\underline{B}}{\mu_0} - \underline{M} \\ \nabla \times \underline{H} = \underline{J}_f \end{array}}$$

Hence, combine everything:

$$\boxed{\underline{B} = \frac{\mu_0 \underline{H}}{(1-\chi_B)} = \mu_r \mu_0 \underline{H}}$$

relative permeability $\mu_r = (1-\chi_B)^{-1}$

As differentials:

$$\boxed{\nabla \times \underline{H} = \underline{J}_f \qquad \oint_c \underline{H} \cdot d\underline{l} = I_f}$$

Magnetic Energy Density

Electro static energy density:

$$u_e = \frac{1}{2} \underline{E} \cdot \underline{D}$$

so for magnetism:

$$u_b = \frac{1}{2} \underline{B} \cdot \underline{H}$$

Exam Qs

1) External field $\underline{E}$ applied to MIL dielectric of relative permittivity ε.

a) Explain how polarisation field $\underline{P}$ is induced by E. (How it modifies induced E).

when external field applied to polar dielectric, molecules rotate to produce polarisation field in opposite direction to external field. In non-polar dielectric (+ polar), electron cloud is stretched relative to nucleus to produce polarisation electric field in opposite direction. Field inside is reduced.

Lecture 7

Displacement current

$$I = \frac{dQ}{dt} = \oint_S \underline{J} \cdot d\underline{S} \quad \text{and } Q = \int_V \rho \, dV$$

$$\frac{\partial}{\partial t}\int_V \rho \, dV = \oint_S \underline{J} \cdot d\underline{S}$$

Divergence

$$\frac{\partial}{\partial t}\int \rho \, dV = -\int_V \nabla \cdot \underline{J} \, dV$$

$$\boxed{\frac{\partial \rho}{\partial t} = -\nabla \cdot \underline{J}} \quad \text{Continuity Eq}$$

from previous equations, it shows $\nabla \cdot \underline{J} = 0$, which is wrong!

$$\nabla \times \underline{B} = \mu_0 \underline{J} + \text{displacement current}.$$

$$\mu_0 \nabla \cdot \underline{J} = -\nabla \cdot (\text{displacement current}).$$

Using Gauss' Law

$$\mu_0 \frac{\partial}{\partial t} \varepsilon_0 \nabla \cdot \underline{E} = \nabla \cdot I_d$$

$$I_d = \mu_0 \varepsilon_0 \frac{dE}{dt}$$

Hence

$$\boxed{\nabla \times B = \mu_0 \underline{J} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}}$$

Unit 2: Electromagnetic Waves

Lecture 1

Maxwell's Equations.

$$\nabla \cdot \underline{E} = \frac{\rho_f}{\varepsilon_r \varepsilon_0} \qquad \nabla \cdot \underline{B} = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times \underline{B} = \mu_r \mu_0 J_f + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \varepsilon_r$$

EM is a strong force!

Conductors

free to move charges

free charge density $= \rho_f$

free current density $\underline{J}_f = \sigma \underline{E}$

Dielectrics

$\hat{e}$ dipoles $\underline{p} = q\underline{l}$

polarisation $\underline{P} = \frac{\Sigma \underline{p}}{d\tau}$

$\rho_b = -\nabla \cdot \underline{P}$

bound

$\underline{J}_p = \frac{\partial \underline{P}}{\partial t}$

polarisation

Magnetic

$\hat{m}$ dipoles $\underline{m} = I\underline{A}$.

magnetisation $\underline{M} = \frac{\Sigma \underline{m}}{d\tau}$

$\rho = 0$

$\underline{J}_b = \nabla \times \underline{M}$.

Its not as simple as this e.g. you will have current due to all 3 material types. We need to combine them:

$$\rho = \rho_f + \rho_b = \rho_f - \nabla \cdot \underline{P}$$

$$\underline{J} = \underline{J}_f + \underline{J}_p + J_b = \underline{J}_f + \frac{\partial \underline{P}}{\partial t} + \nabla \times \underline{M}.$$

So we can sub these into Maxwell's equations when needed:

$$\nabla \cdot \underline{E} = \frac{\rho_f - \nabla \cdot \underline{P}}{\varepsilon_0}$$

$$\nabla \cdot (\varepsilon_0 E + \underline{P}) = \rho_f$$

electric displacement

$$\boxed{\underline{D} = \varepsilon_0 \underline{E} + \underline{P}} \Rightarrow \boxed{\nabla \cdot \underline{D} = \rho_f.}$$

Now let's use Ampères' law:

$$\nabla \times \underline{B} = \mu_0 \left(\underline{J}_f + \frac{\partial \underline{P}}{\partial t} + \nabla \times \underline{M} \right) + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\mu_0 \frac{\partial \underline{P}}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = \frac{\mu_0 \partial (\underline{P} + \varepsilon_0 \underline{E})}{\partial t} = \mu_0 \frac{\partial D}{\partial t}$$

$$\nabla \times \left(\frac{\underline{B}}{\mu_0} - \underline{M} \right) = \underline{J}_f + \frac{\partial D}{\partial t} \quad \& \quad \boxed{\underline{H} = \frac{\underline{B}}{\mu_0} - \underline{M}}$$

$$\boxed{\nabla \times H = \underline{J}_f + \frac{\partial D}{\partial t}}$$

HIL Media

to make calculations easier, assume things are HIL media, (Homogeneous, Isotropic, Linear).

Linear means polarisation is proportional to B field:

$$\underline{P} = \varepsilon_0 \chi_E \underline{E}.$$ electric susceptibility, dimensionless.

$$\underline{M} = \chi_m \underline{H}$$ ← H field magnetisation.

so:

$$\boxed{\begin{aligned}\underline{D} &= \varepsilon_0(1+\chi_e)\underline{E} = \varepsilon_r \varepsilon_0 \underline{E} \\ \underline{B} &= \mu_0(1+\chi_m)\underline{H} = \mu_r \mu_0 \underline{H}\end{aligned}}$$

$\varepsilon_r = 1 + \chi_e$ rel permittivity.

in a vacuum ε_r and μ_r are 1. Easy peasy!

For this unit, we'll assume all materials are HIL, so use the new modified Maxwell's Equations at start of prev page. ($J_f = \sigma \underline{E}$ here).

lecture 2

Derivation of the wave Equation

from maths:

$$\underbrace{\nabla \times (\nabla \times \underline{E})}_{\text{Faraday's}} = \underbrace{\nabla(\nabla \cdot \underline{E})}_{=0} - \nabla^2 \underline{E}$$

$$-\nabla \times \frac{\partial \underline{B}}{\partial t} = -\frac{\partial}{\partial t} \nabla \times \underline{B}.$$

$$-\nabla^2 E = \underbrace{\frac{\partial}{\partial t}(\nabla \times \underline{B})}_{\text{Ampère's}} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

3D wave eq for $\underline{E}$ field

$$\boxed{\nabla^2 \underline{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \underline{E}}{\partial t^2}}$$

$$\underbrace{\nabla \wedge (\nabla \wedge B)}_{\text{Ampère}} = \underbrace{\nabla(\nabla \cdot \underline{B})}_{=0 \text{ monopole.}} - \nabla^2 \underline{B}$$

$$\mu_0 \varepsilon_0 \nabla \times \frac{\partial \underline{E}}{\partial t} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \wedge E)$$

$$-\nabla^2 \underline{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \underline{E}) = -\varepsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

$$\boxed{\nabla^2 \underline{B} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}}$$ 3D wave eq for B field

Lecture 2

Make them into 1 equation:
in 1D:

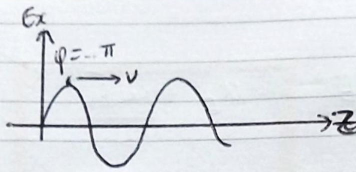
$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

looks like $\boxed{\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}}$

with $v^2 = \frac{1}{\mu_0 \varepsilon_0}$

its easier to manipulate these assuming they're loads of sines & cosines:

Plane Sinusoidal waves.



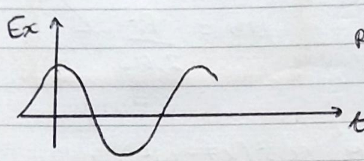
$$E_x(z,t) = E_0 \cos(\underbrace{kz - \omega t}_{=\varphi})\hat{x}$$

amplitude, $V m^{-1}$

space variation

time variation

freq $\nu = \frac{\omega}{2\pi}$



period $\tau = 1/\nu$
$= 2\pi/\omega$

k = wavenumber $= 2\pi/\lambda$

Phase speed $v = \omega/k = \nu\lambda$ (freq)

if $\varphi = kz - \omega t$, $z = \frac{\omega}{k}t + \frac{\varphi}{k}$

Verify This Equation Is Valid

Sub E_x into the 1D wave equation:

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$E_x = E_0 \cos(kz - \omega t)$

$\frac{\partial^2 E_x}{\partial z^2} = -k^2 E_0 \cos(kz - \omega t)$

$\frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_0 \cos(kz - \omega t)$

$\frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_x$ $\qquad$ $\frac{\partial^2 E_x}{\partial t^2} = -k^2 E_x$

$$-k^2 E_x = \mu_0 \varepsilon_0 \omega^2 E_x$$

$$k^2 = \mu_0 \varepsilon_0 \omega^2$$

$$\boxed{\frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c}$$

wow!

lecture 3

Wave Polarisation

$$\underline{E}(z,t) = E_0 \cos(kz - \omega t)\,\hat{x}$$

Faraday's law $\Rightarrow$ $\nabla \wedge E = -\frac{\partial B}{\partial t}$ $\quad \nabla \times E = \frac{\partial E_x}{\partial z}\hat{y} = -kE_0 \sin(kz-\omega t)\hat{y}$

$$\frac{\partial B}{\partial t} = -\nabla \wedge E = kE_0 \sin(kz-\omega t)\hat{y} \Rightarrow \underline{B}(z,t) = \frac{E_0}{c}\cos(kz-\omega t)\,\hat{y}$$

Energy Density

$$u_E = \frac{\underline{E}\cdot\underline{D}}{2} \qquad \underline{D} = \varepsilon_0 \underline{E}$$

$$u_B = \frac{\underline{B}\cdot\underline{H}}{2} \qquad \underline{H} = \frac{\underline{B}}{\mu_0}$$

$$\boxed{u_E = \frac{\varepsilon_0 E^2}{2} \qquad u_B = \frac{B^2}{2\mu_0}}$$

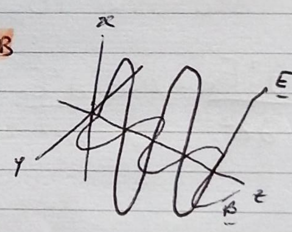
sub these in

$$u_E = \frac{\varepsilon_0 E_0^2}{2}\cos^2(kz-\omega t)\hat{x} \qquad u_B = \frac{E_0^2}{2\mu_0 c^2}\cos^2(kz-\omega t)\hat{y}$$

$$= \frac{\varepsilon_0 E_0^2}{2}\cos(kz-\omega t)$$

Hence $u_E = u_B$

Total energy $= u_E + u_B$



Monopole law $\nabla\cdot\underline{B} = 0$
$\nabla\cdot\underline{E} = 0$ in a vacuum.

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial z} + \cdots = 0 \qquad \cdots + \frac{\partial B_z}{\partial z} = 0$$

but since $\frac{\partial}{\partial x} = 0$ $\quad \frac{\partial}{\partial y} = 0$ for both (look at diagram)

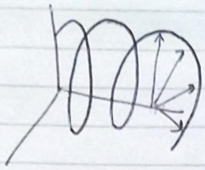
that means $\frac{\partial E_z}{\partial z}$ & $\frac{\partial B_z}{\partial z} = 0$

but that means we can't have any part of wave parallel to propagation. EM waves can't be longitudinal.

Let's now say the wave has $\underline{E}$ & $\underline{B}$ with >1 components of x, y, z

$\underline{E} = E_{0x} \cos(kz-\omega t)\hat{x} + E_{0y} \cos(kz - \omega t)\hat{y}$ Same for $\underline{B}$.

Circularly Polarised waves



$E(z,t) = E_0 \cos(kz-\omega t)\hat{x} - E_0 \sin(kz-\omega t)\hat{y}$

So now $\hat{x}$ & $\hat{y}$ out of phase by $\pi/2$.

The above is clockwise/right-handed

left handed/anticlockwise $\hat{x} + \hat{y}$ instead of $\hat{x} - \hat{y}$.

In any Direction:

waves don't just travel in z, they travell everywhere.

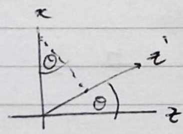
$$\boxed{\underline{E}(\underline{r},t) = \underline{E_0} \cos(\underline{k}\cdot\underline{r} - \omega t) \qquad |\underline{k}| = k \leftarrow \text{wave vector} \qquad r = (x,y,z) \qquad \underline{E_0} = (E_{0x}, E_{0y}, E_{0z}) \perp \text{ to } \underline{k}}$$

$\hat{\underline{k}}$ is direction of propagation.

$$|k| = \frac{2\pi}{\lambda}$$

Lecture 4

if the wave propagated at an angle to x:



$\underline{E}(z',t) = E_0 \cos(z'k - \omega t)$

$z' = z\cos\theta + x\sin\theta$.

Hence: $\underline{E}(x,z,t) = \underline{E_0}\cos((kz\cos\theta + kx\sin\theta) - \omega t)$.

So $\underline{K} = (k_x, 0, k_z) = k(\sin\theta, 0, \cos\theta)$

$\underline{E} = E_0 \cos((k_x x + k_z z) - \omega t) = E_0 \cos(\underline{k}\cdot\underline{r} - \omega t)$.

proof of above eqs.

for B field: $|B_0| = \frac{|E_0|}{c}$

$$\boxed{\begin{aligned} \underline{E}(r,t) &= E_0 \cos(\underline{k} \cdot \underline{r} - \omega t) \\ \underline{B}(\underline{r},t) &= B_0 \cos(\underline{k} \cdot \underline{r} - \omega t) \end{aligned}}$$

$|\underline{k}| = 2\pi/\lambda = k$

Introduce DeMoivre's theorem:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\boxed{\underline{E}(r,t) = E_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)}}$$

So now when we want to differentiate $\underline{E}$ or $\underline{B}$:

$f(x) = e^{ax}$ $\quad \frac{\partial f}{\partial x} = a e^{ax}$ $\quad = a f(x)$.

Hence: $\boxed{\frac{\partial^n}{\partial t^n} \underline{E} = (-i\omega)^n \underline{E}}$ $\quad$ so $a = -i\omega$. like algebra.

And the operator ∇ is just $i\underline{k}$ $\quad \nabla = i\underline{k}$

$$\nabla \cdot \underline{E} = i\underline{k} \cdot \underline{E}$$

$$\nabla \wedge \underline{E} = i\underline{k} \wedge \underline{E}$$

$$\nabla^2 \underline{E} = -\underline{k}^2 \underline{E}$$

proof in lecture 4 slides.

so $\nabla^2 \underline{E} = -k^2 \underline{E} = -\mu_0 \varepsilon_0 \omega^2 \underline{E} = \frac{\omega}{k} = c$ cool!

Vacuum
$$\left\{ \begin{aligned} & i\underline{k} \cdot \underline{E} = 0 \implies \underline{k} \cdot \underline{E} = 0 \\ & i\underline{k} \cdot \underline{B} = 0 \implies \underline{k} \cdot \underline{B} = 0 \\ & i\underline{k} \wedge \underline{E} = i\omega \underline{B} \implies \underline{B} = \frac{\hat{\underline{k}} \wedge \underline{E}}{c} \end{aligned} \right.$$

$\underline{E}$ & $\underline{B}$ $\perp$ $\underline{k}$.

$$i\underline{k} \wedge \underline{B} = -i\mu_0 \varepsilon_0 \omega \underline{E} \implies \underline{E} = -c\hat{\underline{k}} \wedge \underline{B}.$$

wow! So $\underline{E}$ & $\underline{B}$ $\perp$ $\underline{k}$ & equal U_E U_B proof.

Circular Waves in Complex

$$\underline{E}(\underline{r},t) = E_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)} \hat{\underline{x}} + E_0 e^{i(\underline{k} \cdot \underline{r} - \omega t \pm \pi/2)} \hat{\underline{y}}$$

sin

EM waves in a HIL medium

for a dielectric, $\underline{J}_f = 0$ $\rho_f = 0$ non-conducting

only Ampere-Maxwell equation changes.

$$\nabla \cdot E = 0 \qquad \nabla \cdot B = 0$$

$$\nabla \wedge \underline{E} = -\frac{\partial B}{\partial t} \qquad \nabla \wedge B = \mu_r \mu_0 \varepsilon_r \varepsilon_0 \frac{\partial E}{\partial t}.$$

Hence,

$$\nabla^2 E = -k^2 E = \mu_r \mu_0 \varepsilon_r \varepsilon_0 \omega^2 E = \boxed{v = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_0 \varepsilon_r}}}$$

$$n = \frac{c}{v} = \sqrt{\mu_r \varepsilon_r} \qquad B = \frac{\hat{\underline{k}} \wedge \underline{E}}{v} \qquad E = -v \hat{\underline{k}} \wedge \underline{B}$$

c swaps to v.

$E \perp B$ still, & $u_E = u_B$ still.

So not much has changed!
For a conducting medium, this changes drastically.

Lectures

Energy in inductors & Capacitors

in an inductor

$$\text{Work} = \int I V dt = \int I L \frac{dI}{dt} dt = \frac{1}{2} L I^2$$

Eenergy

inductance

Energy goes into magnetic field.

Eg in solenoid $B = \mu_0 \mu_r N I$

$$L = \mu_r \mu_0 N^2 \pi r^2 l$$

for HIL medium $u_B = \dfrac{B^2}{2\mu_r \mu_0}$

So the $\frac{1}{2} L I^2$ becomes $\dfrac{\mu_0 \mu_r N^2 \pi r^2 l}{2} I^2$

$$= \frac{\mu_r \mu_0 N^2 \pi r^2 l}{2 \mu_r^2 \mu_0^2 N^2} B^2$$

area x L = volume

$$\text{Energy} = \frac{\pi r^2 l B^2}{2\mu_0 \mu_r} = \frac{B^2}{2\mu_0 \mu_r} \text{volume} = u_B {}^* \text{Vol}$$

= exactly what we expect ✓

Now for Capacitor

$$\text{Energy } W = \int I V \, dt = \int \frac{dQ}{dt} V \, dt = \int C V \frac{dV}{dt} \, dt = \tfrac{1}{2} C V^2$$

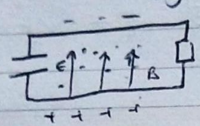
$$u_E = \frac{\varepsilon_r \varepsilon_0 E^2}{2} \qquad \tfrac{1}{2} C V^2 = \tfrac{1}{2} C d^2 E^2 = \frac{\varepsilon_r \varepsilon_0 A d^2 E^2}{2d}$$

W stored in e field between capacitor plates.

$$= \frac{\varepsilon_r \varepsilon_0 E^2}{2} \text{ Volume}$$

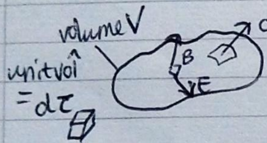
= Exactly what we expect ✓

So EM energy can be stored in fields in air.



Poynting's Theorem

Imagine Electromagnetic potato



$$\text{Power} = \underset{\text{mass}}{m} \underset{\text{velocity}}{\underline{v}} \cdot \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\frac{m \underline{v} \cdot \underline{v}}{2}\right)$$

$$\text{Energy } W = \int_V d\tau \left(\frac{\underline{E} \cdot \underline{D} + \underline{B} \cdot \underline{H}}{2}\right) \qquad = \frac{d}{dt}\left(\tfrac{1}{2} m v^2\right) = q \underline{v} \cdot (\underline{E} + \underline{v} \wedge \underline{B})$$

$$* \quad \frac{\partial W}{\partial t} = -\int_V d\tau \, \underset{\text{rate of change of energy}}{R} - \oint_S \underset{\text{total energy going in/out of system}}{\underline{S} \cdot d\underline{a}}$$

$$\boxed{\text{Power} = q \, \mathbf{\underline{v}} \cdot \mathbf{\underline{E}}}$$

Energy lost by accelerating particles per particle

$$\underline{F} = q(\underline{E} + \underline{v} \wedge \underline{B}) = m \frac{d\underline{v}}{dt}$$

$$\underline{F} \cdot \underline{v} = \text{Power } P = q \underline{v} \cdot \underline{E}. \quad \text{so:} \quad P_e = -e \underline{v}_e \cdot \underline{E}$$

total energy changing time vol: $R = n_e P_e = -\underbrace{e n_e \underline{v}_e}_{\text{flux of charge = current density}} \cdot \underline{E}$

Hence

$$\boxed{R = \underline{J}_{free} \cdot \underline{E}}$$

SEE LECTURE 5 SLIDES !!!

$$\oint_S \underline{S} \cdot d\underline{a} = -\frac{\partial W}{\partial t} - \int_V d\tau R = \ldots\ldots$$

$\underline{E} = \frac{\underline{D}}{\varepsilon_r \varepsilon_0}$ $\quad \underline{H} = \frac{\underline{B}}{\mu_r \mu_0}$ $\quad \boxed{\oint_S \mathbf{S} \cdot d\mathbf{a} = \int_V d\tau (\underline{H} \cdot \nabla \wedge \underline{E} - \underline{E} \cdot \nabla \wedge \underline{H})}$ yay !!

using vector identity: $\nabla \cdot (\underline{F} \wedge \underline{G}) = \underline{G} \cdot \nabla \wedge \underline{F} - \underline{F} \cdot \nabla \wedge \underline{G}$

Hence $\oint_S \mathbf{S} \cdot d\mathbf{a} = \oint_S (\mathbf{E} \wedge \mathbf{H}) \cdot d\mathbf{a}$

$$\mathbf{S} = \mathbf{E} \wedge \mathbf{H}$$

for HIL medium: $\boxed{\mathbf{S} = \frac{\mathbf{E} \wedge \mathbf{B}}{\mu_r \mu_0}}$

$\mathbf{S}$ = Poynting's vector
- Points in direction of energy flow. $|\mathbf{S}|$ mag of flow.

Hence:

$$\frac{\partial}{\partial t}\left(\frac{\mathbf{E} \cdot \mathbf{D}}{2} + \frac{\mathbf{B} \cdot \mathbf{H}}{2}\right) + \nabla \cdot \mathbf{S} + \mathbf{J}_f \cdot \mathbf{E} = 0$$

wow !!

In steady state:

$$\oint_S \mathbf{S} \cdot d\mathbf{a} + \int_V \mathbf{J}_f \cdot \mathbf{E} \, d\tau = 0$$

so if $\mathbf{J}_f \cdot \mathbf{E} >$, then $\nabla \cdot \mathbf{S} < 0$. Sink
eg Resistor

$$\nabla \cdot \mathbf{S} + \mathbf{J}_f \cdot \mathbf{E} = 0$$

if $\mathbf{J}_f \cdot \mathbf{E} <$, then $\nabla \cdot \mathbf{S} > 0$. Source
eg Battery

if $\mathbf{J}_f \cdot \mathbf{E} = 0$, then $\nabla \cdot \mathbf{S} = 0$, flux into $\mathbf{S}$ = flux out
eg. Vacuum.

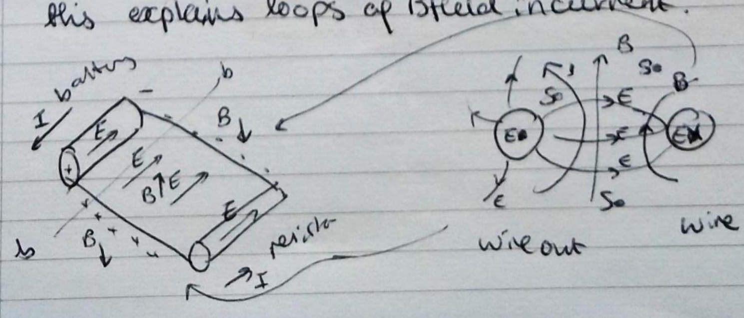
no currents, no charge flow.

Lecture 6 Electromagnetic Energy flow

using above eqn $\oint_S \underline{S} \cdot d\underline{a} + \int_V \underline{J}_f \cdot \underline{E} \, d\tau = 0$

$$\nabla \cdot \underline{S} + \underline{J}_f \cdot \underline{E} = 0.$$

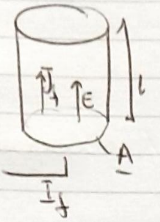
this explains loops of B field in current.

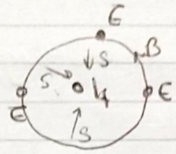


Poynting flux !!!!

S ⊙ out of page.
for b – b line

In the resistor.





$$\text{Power } P = \int_V \underline{J}_f \cdot \underline{E} \, dt$$

$$= \underline{J}_f \, \underline{E} \, \underline{A} l$$

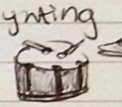
but $\underline{J}_f \underline{A} = I_f$

$\underline{E} l = V$ so $P = I_f V$ wow!

S is Poynting flux. STOP forgetting !!! :'(

using cylindrical coordinates. $B_\varphi = \dfrac{\mu_0 I_f}{2\pi r}$

$$\underline{S} = \underline{E} \wedge \underline{H} = \frac{\underline{E} \wedge \underline{B}}{\mu_0} = -\frac{E B_\varphi}{\mu_0} \hat{\underline{r}}$$ ← -ve since poynting inwards



hence $P = \dfrac{E I_f}{2\pi r} \times 2\pi r l = P = \underline{E I_f l}$

$V = \underline{E} l$ hence $P = I_f V$

$$\underline{S} = \frac{\underline{E} \wedge \underline{B}}{\mu_r \mu_0} = \frac{E_0 B_0}{\mu_r \mu_0} \cos^2(kz - \omega t) \, \hat{\underline{x}} \wedge \hat{\underline{y}}$$

$$= \sqrt{\frac{\varepsilon_r \varepsilon_0}{\mu_r \mu_0}} \, E_0^{\,2} \cos^2(kz - \omega t) \, \hat{\underline{z}}$$

Thus

$$\boxed{\underline{S} = v \, u_{total} \, \hat{\underline{z}}}$$

$u_{total} = u_E + u_B \, \hat{z}$

$= \varepsilon_r \varepsilon_0 E_0^2 \cos^2(kz - \omega t)$

for example. A light bulb.

the average of $\cos^2 \varphi = 1/2$

so $\langle u_{Tot} \rangle = \varepsilon_r \varepsilon_0 \dfrac{E_0^2}{2}$ hence $\langle S \rangle = v \langle u_{tot} \rangle \hat{\underline{z}}$

"average"

$$\boxed{\langle \underline{S} \rangle = \frac{1}{2} \sqrt{\frac{\varepsilon_r \varepsilon_0}{\mu_r \mu_0}} \, E_0^{\,2} \, \hat{\underline{z}}}$$

Radiation Pressure

EM waves apply momentum to things.

Pressure = F/A
= momentum/Area s^{-1}

Mass density in wave:

$$M = \frac{\langle u_{tot} \rangle}{c^2} \quad kg\,m^{-3}$$

$E=mc^2$

$$F = c\frac{\langle u_{tot} \rangle}{c^2} = Mc \quad kg\,m^{-2}\,s^{-1}$$

$$\alpha \quad \boxed{P_{radiation} = \langle u_{tot} \rangle}$$

So radiation pressure is simple :)

If wave is reflected, it's $2 \times \langle u_{tot} \rangle$.

If we were talking about particle photons

$E = \hbar\omega$ momentum $p = \hbar k$ No. $\gamma^{-1} = nc$

$$\boxed{\begin{aligned} \text{Pressure} &= \hbar k n c \\ &= \hbar n \omega = nE \end{aligned}}$$

$c = \omega/k$

lecture 7

Wave Propagation in an electrical Conductor

$$\boxed{\underline{S} = v\, u_{tot}\, \hat{\underline{z}}}$$

Tim thinks we did this, but no.

σ is electrical conductivity. (Ωm^{-1}) $\underline{J}_f = \sigma \underline{E}$

$\underline{J}_f$ # to $\hat{\underline{k}}$. current in $\hat{\underline{x}}$, parallel to $\underline{E}$ wave amplitude falls over distance.

Maxwell's Equations for H/L Conductor.

$\nabla \cdot \underline{E}$ ← same as vacuum → $\nabla \cdot \underline{B} = 0$

$$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\nabla \wedge \underline{B} = \mu_r \mu_0 \left(\sigma \underline{E} + \varepsilon_r \varepsilon_0 \frac{\partial \underline{E}}{\partial t} \right)$$

Conducting current

disp current

So if disp current >> conduction current, wave is like in a vacuum, but amplitude drops w/ distance. material = "lossy dielectric"

If conduction current >> disp current, wave is strongly altered. its a "good conductor".

R = conduction current ÷ disp current Amplitudes

...

$= \frac{\sigma \{\text{Amplitude of } \underline{E}\}}{\varepsilon_r \varepsilon_0 \omega \{\text{amplitude of } \underline{E}\}}$ $\boxed{R = \frac{\sigma}{\varepsilon_r \varepsilon_0 \omega}}$

Small ω => large R "good conductor"
large ω => Small R "lossy dielectric"

Let's use Faraday's law

Good Conductors.

$$\nabla \wedge (\nabla \wedge \underline{E}) = \nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E}$$

$$\boxed{\nabla^2 E = \mu_r \mu_0 \sigma \frac{\partial E}{\partial t}}$$

Sub this bitch into $\underline{E}(z,t) = E_0 e^{i(kz - \omega t)} \hat{x}$

$k^2 = i \mu_r \mu_0 \cdot \sigma \omega$ $\cancel{-i k^2}$

$K = (1+i)\sqrt{\frac{\mu_r \mu_0 \sigma \omega}{2}} = (1+i) k_c$. $k_c = \sqrt{\frac{\mu_0 \mu_r \sigma \omega}{2}}$.

lecture 8

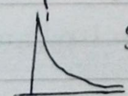
So now:

$\underline{E}(z,t) = E_0 e^{i(kz - \omega t)} \hat{\underline{x}}$ ← propagating in x.

$= E_0 e^{ikz} e^{-i\omega t} \hat{\underline{x}}$

$= E_0 e^{ik_c z} e^{-k_c z} e^{-i\omega t} \hat{\underline{x}}$

$$\boxed{= \underbrace{E_0 e^{-k_c z}}_{\text{Amplitude}} e^{i(k_c z - \omega t)} \hat{\underline{x}}}$$



goes down to approach zero.

Amplitude falls by factor $e^{-1} \approx 0.37$ in distance

$$\boxed{\delta = \frac{1}{k_c} = \sqrt{\frac{2}{\mu_r \mu_0 \sigma \omega}}}$$

skin depth/ attenuation length

$$E_0 e^{-k_c z}$$

dispersive relationship

dispersive phase speed.

$$\boxed{\begin{aligned} V_c &= \frac{\omega}{k_c} = \sqrt{\frac{2\omega}{\mu_r \mu_0 \sigma}} \\ \lambda &= \frac{2\pi}{k_c} = 2\pi\delta \end{aligned}}$$

$$e^{i(k_c z - \omega t)}\hat{\underline{x}}$$

for example:

Cu at 1×10^{6} Hz $\quad V_c \sim 400\,m^{-s}$

$\quad 10\times10^{9}$ Hz $\quad V_c \sim 40000\, ms^{-1}$

Cu at 1×10^{6} Hz $\quad \lambda_c \sim 4\times10^{-4}$ m $\quad \delta = 40\times10^{-5}$m

Cu at 10×10^{9} Hz $\quad \lambda_c \sim 4\times10^{-6}$ m. $\quad \delta = 7\times10^{-4}$ m.

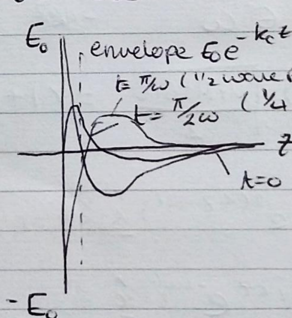
the wave is strongly attenuated by ohmic dissipation. in 1 wavelength, the amplitude falls by factor:

$$\boxed{e^{-k_c \lambda_c} = e^{-2\pi} \approx \frac{1}{500}}$$

δ is distance where amplitude falls ~~to~~ to 37%

the real part of $\underline{E} = E_0 e^{-k_c z} \cos(k_c z - \omega t)\,\hat{x}$

for different times:

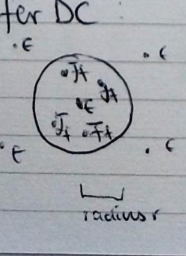


each line will ~~mo~~ move along E_0 axis, (basically flip) and $t=0$ will slowly change to $t = \frac{\pi}{2\omega}$ and keep going.

The Skin Effect.

normally we neglect wire resistance.

for DC:

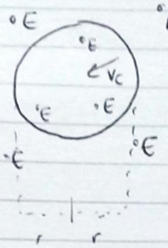


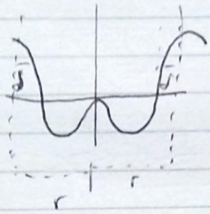
conductivity σ. $\quad E \approx$ uniform. $\quad \underline{J}_f = \sigma \underline{E}$

$$I_f = \pi r^2 \sigma \underline{E}. \quad V = \underline{E}\, L(\text{length}) \quad \frac{V}{I_f} = \boxed{R_0 = \frac{L}{\pi r^2 \sigma}}$$

for AC

Efield alongwire





Efieldoscillates at required frequency. ChangingE propagates into wire from outsideatspeed v_c.

$$\boxed{\begin{aligned} R &= \frac{1}{2\pi r}\sqrt{\frac{\mu_r \mu_0 \omega}{2\sigma}} \\ R &= \frac{1}{2}\frac{r}{\delta}R_0 \leftarrow \text{from DC} \end{aligned}}$$

← lookslike δ !

forawire of givenr at high frequency this becomes

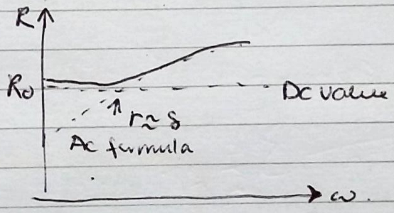
$$\boxed{\delta \propto \frac{1}{\sqrt{\omega}}}$$

high AC only { So there's nopoint inawire with largeradius if skin depth is tiny (inmostsituations)

Thats why we use lotsof thin wiresbundles insteadofa big chunky one.

let's graph it!

for low ω, δ>r uniformE and so $R \simeq R_0$
for high ω, δ<r $E \sim E_{outside}$ only with δ of the surface, AC case



feedback Pleasekeep telling snarky witicisms & funny jokesTim. His spokenlanguage makes it fairly easy to understandthe topic.