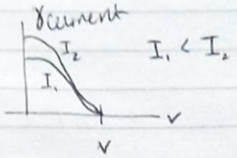
This requires further reading.

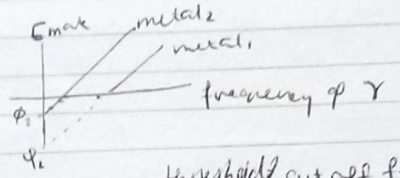
Unit 3: Quantum Mechanics

Lecture 1

Photoelectric Effect



$I_1 < I_2$



$E_{max} = \hbar f - \varphi$

$\hbar = \frac{h}{2\pi}$.

metal work function

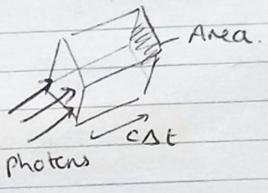
threshold/cutoff frequency when $\hbar f = \varphi$.

when energy drops lower than factors, no e^- released, So light is in discrete packets.

$f = \nu = \omega$ in books lectures ect.

Force = Pressure × Area

$$= \frac{N \hbar f}{c \Delta t A} A$$



$F = \Delta p / \Delta t$

$\Delta p = p$ if γ absorbed.

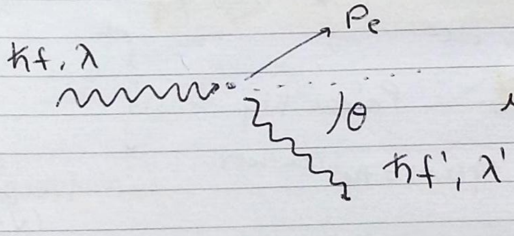
$p_{total} = F\Delta t = \frac{N\hbar f}{c}$

$$\boxed{p = \hbar k = h/\lambda}$$

wow it's de Broglie! wavelength matt loves pronouncing his name correctly!! ''

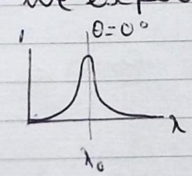
Compton Effect

Even Mervyn can't say it, but he's better than matt.



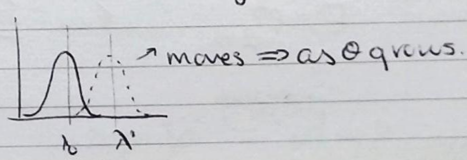
where $\boxed{\lambda' = \lambda + \frac{2\pi\hbar}{mc}(1 - \cos\theta)}$

we expect



most expected

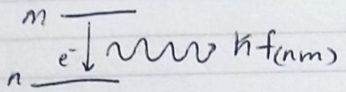
when we change θ



Concrete evidence for γ particles.

Discrete Spectral lines

these show e^- bands have discrete energies
Atoms have minimum groundstate energy.



$$f_{mn} = 2\pi R_0 c \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

So atoms have certain discrete energies.

Electron's DeBroglie wavelengths.

de Broglie thought if γ are particles, β^- can be waves.

$$p = \hbar k \;,\; E_{e^-} = \hbar \omega \text{ or } \hbar f.$$

$k = 2\pi/\lambda$

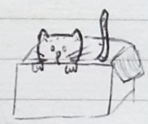
$p = h/\lambda \qquad E = p^2/2m = \tfrac{1}{2} m v^2$

Feedback — Merugn needs to learn to pronounce de Broglie and Huygens.

lecture 2

Schrödinger Equation

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + \underset{\text{Potential}}{V(x,t)}\, \psi.$$



$P(x,t) = |\psi(x,t)|^2$; Probability density.

for wave-like solutions $\psi(x,t) = Ae^{i(kx-\omega t)}$ with free particle. $(V=0)$

the correct wave eq^n must satisfy:

$E = p^2/2m \qquad \hbar\omega = \frac{\hbar^2 k^2}{2m} \qquad E = \hbar f \qquad p = \hbar k$

if you try a variation Schrödingers eq^n it may not work eg $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}$

$-k^2\psi = \frac{1}{c^2}(-\omega^2)\psi$

try the form above, but no $V(x,t)\psi$.

$k^2 \propto \omega^2$ not true!

If particles aren't free ($V \neq 0$) just guess, my guy

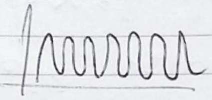
Heisenberg's Uncertainty Principle

$$\boxed{\begin{aligned} \Delta x \Delta p &\geq \tfrac{1}{2}\hbar \\ \Delta t \Delta E &\geq \tfrac{1}{2}\hbar \end{aligned}}$$

So you can't know position & momentum & time & energy very well simultaneously
scale.

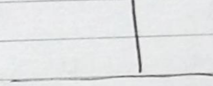
product of uncertainties more than $\frac{1}{2}\hbar$.

lets look at one waveform:



fourier transform $\Rightarrow$

momentum space

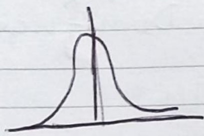


only 1x, 1 momentum component

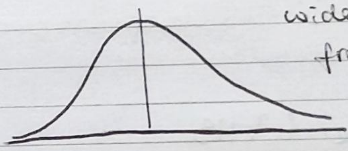
eg $f(x) = \sum_{1}^{\infty} C_n e^{ik_n x}$ $\Rightarrow f(x) = e^{ikx}$

fourier expansion of

So momentum space widens wavefunction from position space.



FT $\Rightarrow$



Example

$\Delta x = 0.1 \times 10^{-10}$

$m = 1.7 \times 10^{-27}$

$\Delta t = 1s.$

$\Delta v = \frac{\hbar}{2 \Delta x m}$

$\Delta x_{new} = t \Delta v$

$\Delta x_{new} \sim 3km$. bruh!

Time Independant Schrödinger Equation

So $V(x)$ no t.

eigenvalue

$$-\frac{\hbar^2}{2m}\frac{\partial^2 u(x)}{\partial x^2} + V(x)\, u(x) = E u(x)$$

look! It's an eigenfunction

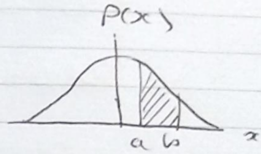
$$\Psi(x,t) = \phi(t)\, u(x) = e^{-i\omega t} u(x).$$

eigenfunction = stationary state

If $V(x,t) = V(x)$, then $\Psi(x,t) = e^{-i\omega t} u x$

$P(x) = |e^{-i\omega t} u(x)|^2 = \underbrace{|e^{-i\omega t}|}_{=1} |u(x)|^2$

$P(x) = |u(x)|^2$. $P(x)$ units 1/length.



total probability

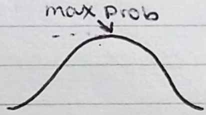
$$P(a \leq x \leq b) = \int_a^b P(x)\,dx$$

where $\int_{-\infty}^{\infty} P(x)\,dx = 1$. wow!

If $u(x)$ is a solution, $A \cdot u(x)$ is also one.

$|A|^2 \int_{-\infty}^{\infty} |u_1(x)|^2\,dx = 1 \qquad |A|^2 = 1 / \int_{-\infty}^{\infty} |u_1|^2\,dx$.

for a simple peak

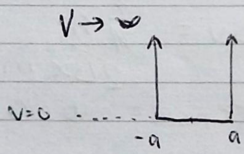


$\frac{dy}{dx} = 0$ is highest probability. Don't forget!!

Lecture 3

Infinite Wells

Infinite square wells.



$V = 0, \quad -a \leq x \leq a$

$V = \infty \quad |x| > a$.

wavefunction

if $x \leq -a$ or $\geq a$ then $u = 0$ particles don't like high V or Energy

If $|x| \leq a$ $V(x) = 0$ then

$$-\frac{\hbar^2}{2m} \frac{\partial^2 u}{\partial x^2} = E u \qquad \frac{\partial^2 u}{\partial x^2} = \left(\frac{-2mE}{\hbar^2}\right) u$$

hence from ODEs $u(x) = A\cos kx + B\sin kx$

$$k^2 = \frac{2mE}{\hbar^2}$$

check: $u' = -kA\sin kx + kB\cos kx$
$u'' = -k^2A\cos kx - k^2B\sin kx.$
$= -k^2 u$ ← that's our original eq.

$$\left(E = \frac{\hbar^2 k^2}{2m} = p^2/2m \right)$$

at $x = a$, u is continuous.

So $u(a) = 0 = A\cos(ka) + B\sin(ka)$. ①

at $x = -a$ u is continuous
$u(-a) = 0 = A\cos(k-a) + B\sin(-ka)$ ②

hence, solve as simultaneous equations.

$2A\cos(ka) = 0$ either $A = 0$ or $\cos(ka) = 0$
$2B\sin(ka) = 0$ $B = 0$ $\sin(ka) = 0$

go back to $u(x)$ at top. if $B = 0$ $\cos(ka)$ must also
$A = 0$ $\sin(ka) = 0$.

for $k = \frac{n\pi}{2a}$ $n = 1, 3, 5, 7 \ldots$

$$\boxed{u(x) = \frac{1}{a^{1/2}} \cos kx}$$

and $\boxed{u(x) = \frac{1}{a^{1/2}} \sin kx}$ for $n = 2, 4, 6 \ldots$

as n increases, energy & groundstate increase.

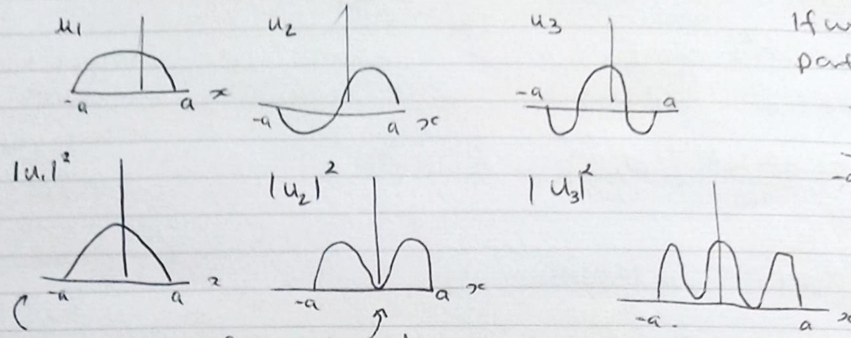
$$\boxed{E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{8ma^2}}$$

discrete energy levels.

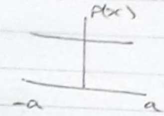
they aren't atomic ones, just wavefunction energy levels.

Thus
$u(x) = A\cos\frac{n\pi x}{2a}$ if n = odd
$A\sin\frac{n\pi x}{2a}$ if n = even.

this looks like



If we put a classical particle in a well:



now for probability distribution

remember $P(x) = |u|^2 dx$ so can't be zero!

If u is wider, it decreases kinetic energy, but if it the potential is infinite, so it needs to find a balance, which creates the troughs etc.

Normalise wavefunction

just makes sure your distribution summs to 1.

$$\int_{-\infty}^{\infty} |u|^2 dx = 1.$$

but so our function is zero anywhere but between a, a -a. So don't count it in integral:

$$\int_{-a}^{a} |u|^2 dx = 1 = A^2 \int_{-a}^{a} \cos^2\left(\frac{n\pi x}{2a}\right) dx$$

$$1 = A^2 \int_{-a}^{a} \frac{1}{2}\left(1 + \cos\left(\frac{2\pi n x}{2a}\right)\right) dx$$

$1 = A^2 a$ so $u(x) = \frac{1}{\sqrt{a}} \cos\left(\frac{n\pi x}{2a}\right)$

even parity (symmetric) $n = 1, 3, 5$

Same way for the $\sin(x)$ bit.

odd parity $n = 2, 4, 6$

sucks but that's physics!

example

$m_e = 9.11 \times 10^{-31}$

$a = 1 \times 10^{-10}$

Energy separation between ground & first?

$$E = \frac{\hbar^2 \pi^2 n^2}{8ma^2}$$

ground is $n=1$

g. 1st is $n=2$

$G = 1.5 \times 10^{-18}$

$1^{st} = 6 \times 10^{-18}$

$Sep = 4.5 \times 10^{-18} \approx 30 eV$

Example: Salt grain

$m = 10^{-10}$

$a = 10^{-6}$ m

$E_{ground} = E_1 = 10^{-27}$ eV

$E = 10^{-5}$ eV   whats excited state number, n.

$E_n \sim E_{ground} \sim 10^{-5} eV$

$E_n = 10^{-5}$ eV $+ 10^{-27}$ eV.

$\therefore$

$n^2(10^{-27}) = 10^{-5}$. $= 1 \times 10^{11}$.

Lecture 4

Complex functions

$$\boxed{\int_{-\infty}^{\infty} u_n^* u_n \, dx = 1 \qquad \int_{-\infty}^{\infty} |u_n|^2 \, dx = 1}$$

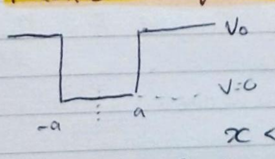
If Wave functions are orthogonal

$$\boxed{\int_{-\infty}^{\infty} u_m^* u_n \, dx = 0 \qquad m \neq n}$$

if wave functions are orthonormal

$$\int_{-\infty}^{\infty} u_m^*(x) u_n(x) \, dx = \delta_{nm} \qquad \begin{matrix} 0 \text{ if } m \neq n \\ 1 \text{ if } m = n \end{matrix}$$

Finite Square Wells



$V_x = 0 \quad |x| \leq a$ $E < V_0$ in bound state.

$V_0 \quad |x| \geq a$

when $|x| \leq a$, $u(x) = A\cos kx + B\sin kx$ $k^2 = 2mE/\hbar^2$

$x < -a \quad u(x) = Ce^{\alpha x}$ $x > a \quad u(x) = De^{-\alpha x}$

$\alpha^2 = 2m(V_0 - E)/\hbar^2 = k_0^2 - k^2$

Even parity with:
$C = D$
$B = 0$
Odd Parity w/:
$C = -D$
$A = 0$

we need discrete k values

at $x = -a$ u is continuous

$$Ce^{-\alpha a} = A\cos ka - B\sin(ka) \quad ①$$

$x = -a$ $\partial u / \partial x$ is continuous.

$\alpha Ce^{\alpha x}$, $u' = -Ak\sin kx + Bk\cos kx$.

$$\alpha Ce^{-\alpha a} = Ak\sin(ka) + Bk\cos(ka) \quad ②$$

$x = a$ $$De^{-\alpha a} = A\cos(ka) + B\sin(ka) \quad ③$$
$$-\alpha De^{-\alpha a} = -Ak\sin(ka) + Bk\cos(ka) \quad ④$$

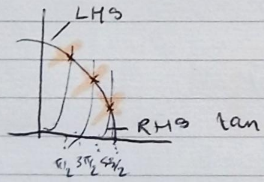
① + ③	$(C+D)e^{-\alpha a}$	$= 2A\cos ka$	⑤
② + ④	$\alpha(C-D)e^{-\alpha a}$	$= 2Bk\cos ka$	⑥
③ − ①	$(D-C)e^{-\alpha a}$	$= 2B\sin ka$	⑦
② − ④	$\alpha(C+D)e^{-\alpha a}$	$= 2kA\sin ka$	⑧

⑧/⑤ = $\alpha = k\tan(ka)$ ← familiar — unless $C+D=0$, $A=0$ odd parity
⑥/⑦ $\alpha = -k\cot(ka)$ unless $C-D=0$, $B=0$ even parity

Sub in $\alpha = \sqrt{k_0^2 - k^2}$ to get k & allowed states.

EVEN PARITY $(k_0^2a^2 - k^2a^2)^{1/2} = ka\tan(ka)$

Can't be done analytically, use *NUMERICAL METHODS*

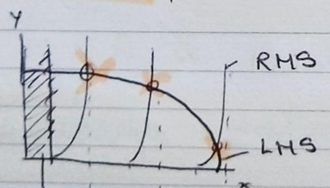


3 discrete values of k and hence E.
even as the well gets smaller, there's always a ground state.
1st state: $ka = 2.6$ $E = \frac{\hbar^2(2.6)^2}{2ma}$
3rd ...

both for $k_0a = 5$

ODD PARITY

... $= -ka\cot(ka)$



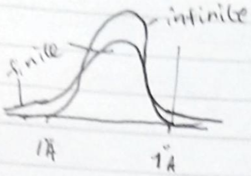
State with no ground

3 discrete values
There is a low enough energy in which there's no bound states.

here $ka = 1.3$ so Ground $E = \frac{\hbar^2(1.3)^2}{2ma}$
2nd state & 4th state
1st & 3rd are even, as above.

Lecture 5

If we put finite & infinite together



$V_0 = 95eV$

infinite stops at $a = 1Å$.

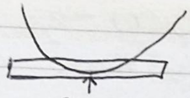
Finite is way shallower, but goes out of well

~~un~~ Narrowing the well maximises energy.

Harmonic Oscillators.

All potentials vary like $V(x) = V_0 + ax^2$ when near a minimum.

eg semiconducting nanotube. most particles stay in centre. Potential increases w/ x^2



Centre - Quantum Dot.

Hooke's Law: $V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$ $\qquad \omega = \sqrt{k/m}$

let $y = x/\left(\frac{\hbar}{m\omega}\right)^{1/2}$ dimensionless so $y = x/x_0$ $\qquad x_0 = \left(\frac{\hbar}{m\omega}\right)^{1/2}$

So we've basically swapped x with y for funsies :)

This will look similar to the wells.

Now let's add Schrödinger's equation: use chain rule $\frac{y}{x} = \frac{u}{y}\frac{y}{x}$

$$-\frac{\hbar\omega}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1}{2}\hbar\omega y^2 u = Eu$$

α must be added.

$$\partial^2 u/\partial y^2 + (\alpha - y^2)u = 0. \qquad \alpha = \frac{2E}{\hbar\omega}$$

Thus $\alpha = 2n+1$ $\qquad E = \frac{\alpha\hbar\omega}{2} = (n + 1/2)\hbar\omega$ $\quad n = 0, 1, 2, 3$

wells, h.o, stuff all go up different energy levels. REMEMBER!!

Ground state = 0 point energy $E_0 = \frac{1}{2}\hbar\omega$.

to solve the $\frac{\partial^2 u}{\partial y^2} + (\alpha - y^2)u = 0$, guess $u = y^n e^{-y^2/2}$ as you do.

$$\frac{\partial^2 u}{\partial y^2} \simeq -(n+1)y^n e^{-y^2/2} + y^{n+2}e^{-y^2/2}$$

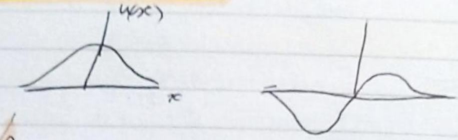
$$\approx y^{n+2}e^{-y^2/2} = y^2 u.$$

which is what we guessed - sort of.

So, the harmonic oscillator wavefunctions are:

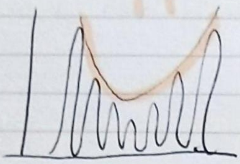
$$u(x) = A_n H_n (x/x_0) e^{-x^2/2x_0^2}$$ H_n are Hermite polynomials.

the first 2 look like

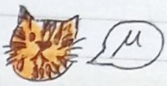


similar to wells.

$n=10$



fun! it's tiger teeth



at these levels, particle behaviour is starting to look continuous & classical.

Correspondance Principle

Lecture 6

recap: for harmonic oscillators $V(x) = \frac{1}{2} m\omega^2 x^2$

Discrete energy elevels $E = (n + \frac{1}{2})\hbar\omega$. $n = 0, 1, 2$

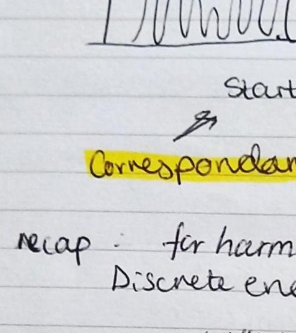
$u_0(x) = \left(\frac{1}{\pi}\right)^{1/4} \left(\frac{1}{x_0}\right)^{1/2} e^{\left(-\frac{x^2}{2x_0^2}\right)}$ E_0 = eigenvalue

$u_1(x) = \left(\frac{4}{\pi}\right)^{1/4}$ " $3/2$ $\cdot x$ "

$u_2(x) = \left(\frac{1}{4\pi}\right)^{1/4}$ same as u_0 but multiply extra $\left(\frac{2x^2}{x_0^2} - 1\right)$

Classical turning point $E_n = \frac{1}{2} m\omega^2 x^2$ zero point $E_0 = \frac{1}{2}\hbar\omega$.

where total Energy = potential energy



classical turning point.

x_c defined by

$$E_n = (n+\tfrac{1}{2})\hbar\omega = \tfrac{1}{2} m\omega^2 x_c^2$$

Example

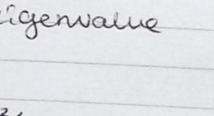
e^- confined by harmonic oscillator. Probability e^- in forbidden region = ?

$$\int_{x_c}^{\infty} |u_0|^2 dx + \int_{-\infty}^{-x_c} |u_0|^2 dx$$

$$2\int_{x_c}^{\infty} |u_0|^2 dx$$

$$1 - 2\int_0^{x_c} |u_0|^2 dx$$

this bit



Quantum: can be represented by all of them

Classical $u(x) = 0$

Expectation values & Operators

$$\langle \hat{O} \rangle = \int_{-\infty}^{\infty} u^*(x)\, \hat{O}\, u(x)\, dx$$
← comp conj

$$\langle x \rangle = \int_{-\infty}^{\infty} x\, |u(x)|^2\, dx$$

Potential $U(x)$ $\langle V \rangle = \int_{-\infty} V(x)\, |u(x)|^2\, dx$

momentum operator $\hat{p} = -i\hbar \frac{\partial}{\partial x}$ $\langle \hat{p} \rangle = \int_{-\infty}^{\infty} u^*(x)\, (-i\hbar)\, \frac{\partial u(x)}{\partial x}$

Kinetic E $\hat{T} = \frac{\hat{p}^2}{2m} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$

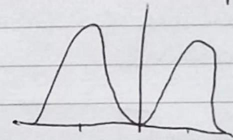
$\hat{p} u = -i\hbar\, N(ik)\, e^{ikx} = \hbar k\, u(x)$
eigenvalue
$u(x)$ is back.

$$\langle \hat{T} \rangle = \int_{-\infty}^{\infty} u^*(x)\, \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} u(x)\, dx$$

if $u(x) = \frac{1}{2\pi} \sum_k a_k e^{ikx}$ $\langle \hat{T} \rangle = \sum_k |a_k|^2 \frac{\hbar^2 k^2}{2m}$.

Example β in 1^{st} excited state $\left(\frac{4}{\pi}\right)^{1/4} \left(\frac{1}{x_0}\right)^{3/2} x\, e^{(-x^2/2x_0^2)}$

whats most probable position?

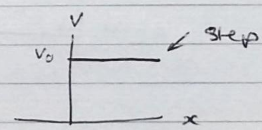


peaks at stationary points

$\frac{\partial |u_1(x)|^2}{\partial x} = 0 = A^2[\text{[illegible]}]$ $u_1 = Axe^{-x^2/2x_0^2}$

$0 = 2x - 2x^3/x_0^2$ $x_0^2 = x^2$. point $x = \pm x_0$

Potential Step



for the case of $E > V_0$

① Solve Schrödinger equation.
So
$u(x) = Ae^{ikx} + Be^{-ikx}$ $x<0$
$u(x) = Fe^{iqx}$ $x>0$
$k^2 = 2mE/\hbar^2$ $q^2 = 2m(E - V_0)/\hbar^2$

$T = \frac{j_F}{j_A}$

$j = \frac{-i\hbar}{2m}\left(u^* \frac{du}{dx} - u \frac{du^*}{dx}\right)$.

② apply boundary conditions
$x=0$, $A + B = F$ $ikA - ikB = iqF$

③ do whatever this is

$R = |B/A|^2$
$T = |F/A|^2\, q/k$
} same R & T as Enfields

$R = \frac{(k-q)^2}{(k+q)^2}$

In the Case $E < V_0$

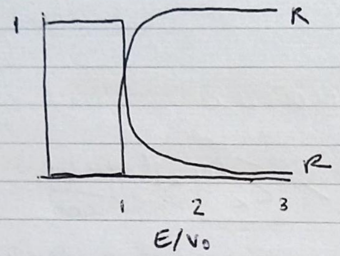
① Same but $u(x) = Ce^{-q'x}$ $x > 0$
② $k^2 =$ Same $q'^2 = 2m(V_0 - E)/\hbar^2$
$q^2 = -q'^2$ $q = iq'$

Boundary Conditions. Same as before

③ $\boxed{R = \frac{(k - iq')^2}{|k + iq'|^2}}$ reduces to 1, wow!

So if $E < V_0$ no travelling waves. all of wave reflects.

lecture 7



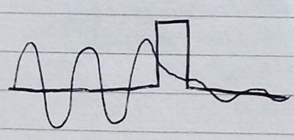
$E > V_0$

$E < V_0$
$R = 1, T = 0.$

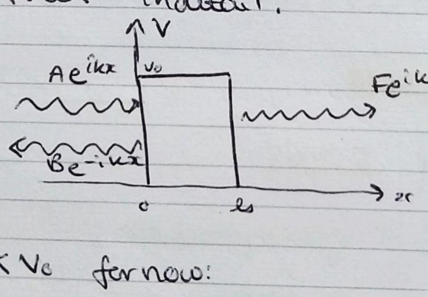
Quantum Tunnelling

probability of particles moving through & out of barrier not impossible

Occurs in Sun. Ammonia decay etc.



let's look at this in detail:



let's look at $E < V_0$ for now:

We want to find transmission probability $T = |\frac{F}{A}|^2$

① write down solutions to Schrödinger
$E < V_0$ where $E = \frac{\hbar^2 k^2}{2m}$ $\alpha^2 = \frac{2m}{\hbar^2}(V_0 - E)$

$u(x) = Ae^{ikx} + Be^{-ikx}$ if $x < 0$
$u(x) = Ce^{\alpha x} + De^{-\alpha x}$ x from 0 to b
$u(x) = Fe^{ikx}$ $x > b$, nothing to reflect there on so no $-ikx$ term.

if x is from 0 to b:

$$\frac{-\hbar^2}{2m}\frac{\partial^2 u}{\partial x^2} + V_0\, u(x) = Eu$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{2m}{\hbar^2}(V_0 - E)\, u$$

$$u = Ce^{\alpha x} + De^{-\alpha x}$$

keep this — x only goes up to $= b$ so the term doesn't grow.

keep this too

Some people write kappa K instead of α, but k and K are very difficult to differentiate visually.

at $x = 0$:

$$A + B = C + D \quad \text{(continuity)} \quad (1)$$

$$ikA - ikB = \alpha C - \alpha D \quad \text{(grad continuity)} \quad (2)$$

at $x = b$

$$Ce^{\alpha b} + De^{-\alpha b} = Fe^{ikb} \quad \text{(continuity)} \quad (3)$$

$$\alpha Ce^{\alpha b} + \alpha De^{-\alpha b} = ikFe^{ikb} \quad \text{(grad continuity)} \quad (4)$$

(1) + (2):

$$2ikA = C(ik + \alpha) + D(ik - \alpha)$$

α(3) + (4):

$$2\alpha Ce^{\alpha b} = (\alpha + ik)\, Fe^{ikb}$$

α(3) − (4):

$$2\alpha De^{-\alpha b} = (\alpha - ik)\, Fe^{ikb}$$

So now:

$$2ikA = \left((ik + \alpha)(ik + \alpha)\frac{e^{-\alpha b}}{2\alpha}Fe^{ikb} - (ik - \alpha)(ik - \alpha)\frac{e^{\alpha b}}{2\alpha}Fe^{ikb}\right)$$

eliminate B, C, D to get T:

$$\frac{F}{A} = \frac{4ik\alpha e^{-ikb}}{(2ik\alpha + \alpha^2 + k^2)e^{-\alpha b} + (2ik\alpha - \alpha^2 + k^2)e^{\alpha b}}$$

This one

if $e^{\alpha b} \gg e^{-\alpha b}$ (large barrier) αb = size/area

$$\left|\frac{F}{A}\right|^2 = \frac{16E(V_0 - E)e^{-2\alpha b}}{V_0^2}$$

it seems the probability has exponential decay.

If there's a large barrier, αb = large

either a large $V_0 \gg k$ or b large or both.

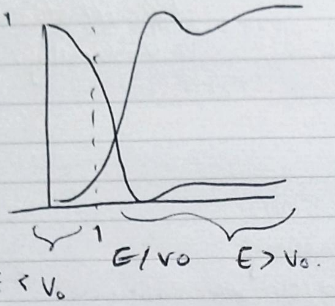
now $T \propto e^{-2\alpha b}$

proportionality

For general Cases:

$E < V_0$ $$T = \left[1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(\alpha b)\right]^{-1}$$

$E > V_0$ $$T = \left[1 + \frac{V_0^2}{4E(E - V_0)} \sin^2(\alpha' b)\right]^{-1} \quad \alpha' =$$



Lecture 8

Many Particle Wavefunctions

N particles: $\psi(x_1, x_2, x_3 \ldots, x_N)$

$$-\frac{\hbar^2}{2m} \sum^N \frac{\partial^2 \psi}{\partial x_i^2} + \sum^N V x_i \, \psi = E\psi$$

or $\psi(x_1, x_2) = U_{n_1}(x_{n_1})(U_{n_2}(x_{n_2})$ so just sub that into the general one.

So that each part $= E_{n_1}$ and E_{n_2}

So $E_{n_1} + E_{n_2} = E$

$$\delta_{n_1 n_2} = \int_{-\infty}^{\infty} u_{n_1}^* u_{n_2} \, dx$$

Linear Combinations

if $\psi(x_1, x_2) = u_i(x_1) \, u_j(x_2)$

$\psi(x_1, x_2) = u_j(x_1) \, u_i(x_2)$ also works

$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}(u_j(x_1) \, u_i(x_2)) \pm (u_i(x_1) \, u_j(x_2))$ also works!

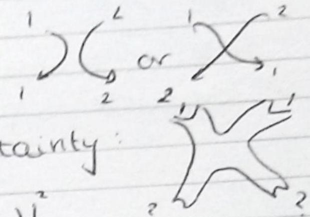
this is most appropriate for indistinguishable particles

(PTO)

Fermion & Boson Wavefunctions

They're indistinguishable.

in the Classical world, particle can go:



but Quantum ones have uncertainty:

For 2 particles we must have $|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$

So we can't just have $u_i x_1, u_j x_2 = \psi(x_1, x_2)$ for indistinguishable particles.

Instead we'll use these:

Fermions	**Bosons**
ψ is antisymmetric (from Pauli Exclusion)	ψ is symmetric
$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}(u_i 1\, u_j 2 - u_i 2\, u_j 1)$	$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}(u_i 1\, u_j 2 + u_i 2\, u_j 1)$
can't be in same state if $i = j$ eg spin.	no constraints.

Eigenvalues & Operators

if u is an eigenstate of operator $\hat{O}$

$$\hat{O}u = \lambda u$$

λ ← Eigenvalue.

Momentum Operator $\hat{P}_x = -i\hbar \frac{\partial}{\partial x}$

Position Operator $\hat{X} = x$ wow!

Hamiltonian $\hat{H} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

Planewaves, e^{ikx}, are Eigenstates to $\hat{H}$ and $\hat{P}_x$.

Examples: above

$\hat{H} e^{ikx} = \frac{\hbar^2 k^2}{2m} e^{ikx} = E e^{ikx}$

$\hat{P}_x e^{ikx} = (\hbar k) e^{ikx} = p e^{ikx}$

Heisenberg's principle's not violated lifetime of State is infinite $\Delta t \to \infty$

$P = \hbar k$

and State is delocalised in space $\Delta x \to \infty$

If wavefunction isn't an eigenstate:

eg $\quad u_0(x) = A e^{\frac{-x^2}{2x_0^2}}$ is for $\hat{H}$ not $\hat{P}_x$

After a measurement of $\hat{P}_x$ the wavefunction will be identicle to a $\hat{P}_x$ eigenstate. Wavefunction changes !!

collapses!

$$\hat{P} u_0 = -i\hbar \frac{\partial u_0}{\partial x} \neq p u$$

Eigenstates of $\hat{P}_x$ are $\quad \phi_n(x) = e^{\frac{ik_n x}{\sqrt{L}}}$

Looking at Harmonic Oscillator

$$u_0(x) = A e^{-x^2/2x_0^2} = \sum_n a_n \phi_n(x)$$

Probability of measuring momentum $\hbar k_m = |a_m|^2$

$$a_m = \int \phi_m^*(x) u_0(x) \, dx$$

So before measurement, the wavefunction is $u_0(x)$

After, it's collapsed to $\phi_m(x)$ w/ probability $|a_m|^2$

Question

from Compton Scattering: $\quad \lambda' = \lambda + \frac{2\pi\hbar}{mc}(1 - \cos\theta)$

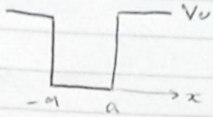
why is there λ'? because some energy is transferred to an electron, So γ energy decreases and λ increases

But they also scatter off the atom (proton) which recoil, but don't move, so large mass = small $\Delta\lambda$

Quantum Mechanics Core Excercises

(2) Finite well

a)



Potential for $n=2$ $= (k_0^2a^2 - k^2a^2)^{1/2} = ka\tan(ka)$ even

$u(x) = De^{-x\sqrt{k_0^2 - k^2}}$ for $>a$

$u(x) = De^{x\sqrt{k_0^2-k^2}}$ for $<a$

Potential $(k_0^2a^2 - k^2a^2)^{1/2} = -ka\cot(ka)$.

b) in well $u(x) = A\sin(kx)$

$u(x)$ cont

$u'(x)$ cont

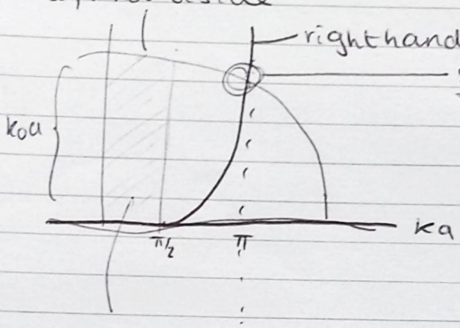
$A\sin(ka) = De^{-x\sqrt{k_0^2-k^2}}$

$Ak\cos(ka) = -\sqrt{k_0^2-k^2}\,De^{-x\sqrt{k_0^2-k^2}}$

c find potential w/no states of odd parity

for $(k_0^2a^2 - k^2a_c^2)^{1/2} = -ka\cot ka$

left hand side



right hand side

Solution but we dont want that

we want this

Since $V_0 = \frac{\hbar^2 k_0^2}{2m}$.

So as k_0a (the radius) gets to $\pi/2$, there'll be no boundstates

$k_0a < \pi/2$

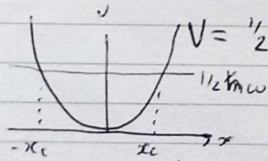
$k_0^2a^2 < \pi^2/4$

$\frac{2mV_0}{\hbar^2}a^2 < \pi^2/4$

$V_0^2 < \frac{\hbar^2\pi^2}{8ma^2}$

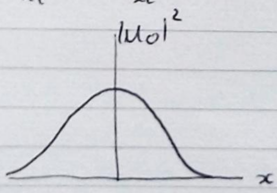
(3)

a)



$E_0 = \frac{1}{2}\hbar\omega$

$u_0 = \left(\frac{1}{\pi x_0^2}\right)^{1/4} e^{\frac{-x^2}{2x_0^2}}$



$E_n = V_x$ $(n+\frac{1}{2})\hbar\omega = \frac{1}{2}m\omega^2x_c^2$

... $x_c^2 = x_0^2 = \hbar/m\omega$

c) $P(x > x_c) = 2\int_{x_c}^{\infty} |u_0|^2\,dx$

$$= 2 \int_{x_0}^{\infty} \left(\frac{1}{\pi x_0^2} \right)^{1/2} e^{-x^2/x_0^2} \, dx$$

$$= 2 \sqrt{\frac{1}{\pi x_0^2}} \int_{x_0}^{\infty} e^{-x^2/x_0^2} \quad \text{or} \quad e^{-u^2} \, du \qquad x/x_0 = u \quad du = 1/x_0 \, dx$$

$$= 2 x_0 \sqrt{\frac{1}{\pi x_0^2}} \left[0.08 \sqrt{\pi} \right] = 0.16$$