

Quantum & Relativity: Special Relativity

Lecture 1

History

- 1676 - Ole Rømer calculated speed of light from Jupiter - light of moons appear faster when closer/approaching. He calculated $10^{-4} c$ (30 km/s) too slow!
- 1849 - Hippolyte Fizeau measured c by reflecting a beam off a distant mirror.
- 1865 - James Clerk Maxwell unified EM & got $3 \times 10^8 \text{ m/s}$

The Principle of Relativity - Galileo

All velocities are relative

$$\vec{r}_B = \vec{r}_A - \vec{v}t$$

↑
nonaccelerating ref frame A
another ref frame

Hence accelerations in each frame are the same:

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A$$

So you can't measure absolute velocities, but you can measure absolute acc.

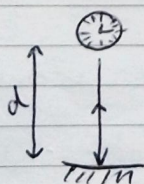
Lecture 2

1905 - Einstein's paper extended Galilean Principal. Dispersed "ether" idea.

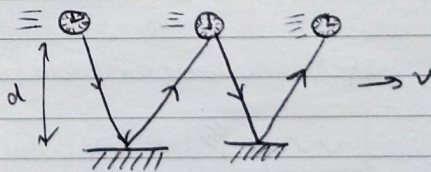
1. laws of physics are identical in all non-accelerating inertial frames.
2. c is a constant for observers in inertial frames (in a vacuum). So it appears constant when you're not accelerating.

Time Dilation

if a clock ticks when a photon bounces between it & a mirror



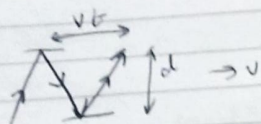
If it moves, the light needs to travel further $\Rightarrow$ clock runs slower $\Rightarrow$ clock sees us running slower



the clock won't realise it's slower, it thinks everything else is slower

The passage of time appears relative here.

let's do maths!



so time between stationary clock:

$$\Delta t = 2d/c$$

time between ticks of moving clocks at v $\Delta t'$:

$$\Delta t' = \sqrt{(2d)^2 + (v\Delta t)^2} / c$$

$$c^2 \Delta t'^2 = (2d)^2 + v^2 \Delta t^2$$

$$(c^2 - v^2) \Delta t'^2 = (2d)^2$$

$$\Delta t'^2 = (c \Delta t)^2 / (c^2 - v^2)$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

so if $v=0$ $\sqrt{1 - \frac{v^2}{c^2}} = 1$
if $v=c$ $\sqrt{1 - \frac{v^2}{c^2}} = 0$
so $\Delta t' \rightarrow \infty$

The Lorentz Factor

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta = \frac{v}{c}$$

$$\text{if } \beta = 0.866$$

$$\gamma = 2$$

he says it too much
form to ignore

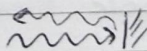
so time dilation can be written as $\Delta t' = \gamma \Delta t$

lorentz factor become significant only at high velocities. non relativistic effects in everyday.

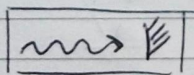
Length Contraction

Imagine measuring length by firing a photon ^{front} to and back and timing flight

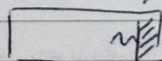
$$2L = ct$$



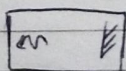
If this experiment were moving, the mirror would also move



> γ looks like it moved more



> γ "moved" less on journey back



outward part of journey return back

$$vt_1' + L' = ct_1' \quad ; \quad L' - vt_2' = ct_2'$$

$$t' = t_1' + t_2' = \frac{L'}{c-v} + \frac{L'}{c+v}$$

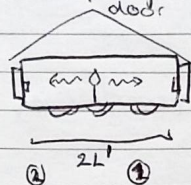
$$= \frac{2L'}{c} \left[\frac{1}{1-v^2/c^2} \right]$$

and:
 $2L = ct$

hence $\Rightarrow$ $L' = \frac{ct'}{\gamma^2} = \frac{ct}{\gamma^2} = \frac{L}{\gamma}$

Another experiment

Train w/ light in middle. Doors w/ light sensor that open.



beam going forward takes longer to travel. no "momentum" principle, & don't do that.

All of this is assuming you're omnipresent.

$$vt_1' + L' = ct_1' \quad ; \quad L' - vt_2' = ct_2'$$

$$\Delta t' = t_1' - t_2' = \frac{L'}{c-v} - \frac{L'}{c+v}$$

Difference in time
between doors

$$= \frac{L'(2v)}{c^2 - v^2} = \gamma^2 \frac{v}{c^2} 2L'$$

Example:

Spaceship 1000m w/ $\beta = 0.866$

① $\gamma = \frac{1}{\sqrt{1-0.866^2}} = 2$ ② length we see $L' = \frac{L}{\gamma} = 500m$

③ clocks look half speed on ship.

④ time dilation between front & back of ship:

$$\Delta t' = \gamma \beta \frac{L}{c} \approx 6 \mu s.$$

lecture 3

Maths of Spacetime.

we can set time as the 4th dimension eg (x, y, z, ct) makes a distance.
 $\uparrow \uparrow \uparrow$
 All have same units now

Spacetime diagrams.

for the purpose of representation, we need to be 2 dimensional or two.
 Anything moving slower than c is here
 worldline of light ray $\rightarrow$ forms light cone.
 faster than speed of light $< 45^\circ$. Forbidden.

A material body will follow a curved path (worldline) but never $< 45^\circ$

ct
 Stationary
 $\} c\Delta t$
 x

t
 moving
 now longer $c\Delta t' = \gamma c\Delta t$
 x

Stretched out by Lorentz factor.

F outframe
 F''
 θ

$\tan \theta = \beta$ from prev lecture.

F''
 c
 reflected F
 look at shorter line. follow the diamond grid

search

hypersurface.

Vectors on the light cone are said to be light like. All events can be contained within. Points outside light cone called space like separated.
 or light separated.

Special Relativity could be understood in a geometrical way 1908 Minkowski

$$\Delta s^2 = c^2 \Delta t^2 - \Delta r^2$$

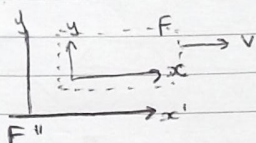
↑
spacetime interval, separated by space & time.

independent of reference frame.

so interval is same for all inertial observers.

The Lorentz Transformation.

Tells us ^{how} separation of time & space change when transferring reference frames (but no acceleration!)



$$\Delta t' = \gamma (\Delta t + v \Delta x / c^2) \quad \text{or} \quad \Delta t = \gamma (\Delta t' - v \Delta x' / c^2)$$

$$\Delta x' = \gamma (\Delta x + v \Delta t) \quad \Delta x = \gamma (\Delta x' - v \Delta t')$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

} v is along x axis.

Consider something stationary on F. $\Delta x = 0$ but not $\Delta x'$
so $\Delta t' = \gamma (\Delta t)$ here.

for length contraction, set $\Delta t' = 0$ for inverse transformation.

$$\Delta x = \gamma \Delta x'$$

we can represent these as matrices.

$$\begin{pmatrix} c \Delta t' \\ \Delta x' \\ \Delta y' \\ \Delta z' \end{pmatrix} = \begin{pmatrix} \gamma & \beta \gamma & 0 & 0 \\ \beta \gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

Velocity Addition Rule.

object in F moving at u . ^{← moving at v.} in frame F', we'd naively expect it u'

$$u' = v + u \quad \text{not true though!}$$

$$\text{Actually: } u' = \frac{\Delta x'}{\Delta t'} = \frac{\gamma (\Delta x + v \Delta t)}{\gamma (\Delta t + v \Delta x / c^2)} \Rightarrow u' = \frac{v + u}{(1 + uv/c^2)}$$

this is so if we had 2 F moving at $0.8c$ & object also $0.8c$ we don't actually get $1.6c$, which is over speed of light!

lecture 4

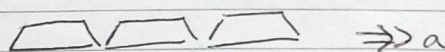
Twin Paradox

1 twin stays on Earth w/ a clock, other twin speeds off on rocket w/ a clock. The both appear slow to the other. The twin returns, and the Earth twin should have aged a lot.

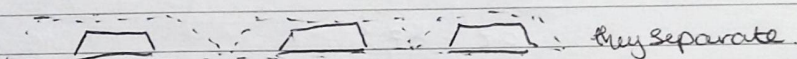
The ship twin hadn't a frame because they accelerated. when she returns around and accelerates, she then sees the Earth twin age like crazy.

Bell's Paradox

but different



3 carriages, selfpropellent, each behind each other. The drivers at the exact same time accelerate until $0.866c$ & what do you expect?



they separate.

they contract and stay in same "position" from driver POV, its still together.

from middle POV, front coach looks faster since γ from clock path has been cut since you're accelerating. looks faster, speedy. behind carriage looks slower, you're stretching path for γ from you to them.

lecture 5

Mass, Energy, Momentum.

Consider this

$$dE = F dx$$

but

$$F = v \frac{dm}{dt}$$

if v is constant.

$$dE = v \frac{dm}{dt} dx = v^2 dm \quad \frac{dx}{dt} = v$$

$$E = v^2 \int dm$$

$$= mv^2$$

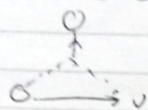
$$E = mc^2$$

relativistic mass m_r

$$E_0 = m_0 c^2$$

rest mass = m_0
mass at $y = 0$

if 2 objects collided, one moving perpendicularly at high speed and the other stationary



to conserve momentum, one must have increased in mass because of time dilation.

$$p = \gamma m_0 v$$

Total energy $E = m_r c^2 = \gamma m_0 c^2$

Squaring gives $E^2 = \frac{(m_0 c^2)^2}{1 - \beta^2} \Rightarrow E^2 = E^2 \frac{v^2}{c^2} + (m_0 c^2)^2$

$$E^2 = (\gamma m_0 v c)^2 + (m_0 c^2)^2 = p^2 c^2 + m_0^2 c^4$$

and Kinetic ~~relativistic~~ energy ^{rest} as:

$$T = (\gamma - 1) m_0 c^2$$

~~total energy~~

Example a clockwork car with $m_0 = 0.1 \text{ kg}$

The car reaches $0.866c$. Assuming no energy lost.

$m_r = ?$ Stays 0.1 kg .

$m_0 = ?$ becomes 0.05 kg ($m_r c^2 = \gamma m_0 c^2$)

why? when we wound the clock up, we gave it lots of energy $\Rightarrow$ increased its rest mass.

Mass can be expressed in GeV/c^2 .

eg $m_p = 1.67 \times 10^{-27} \text{ kg} = 0.938 \text{ GeV}/c^2$

momentum can be written as GeV/c

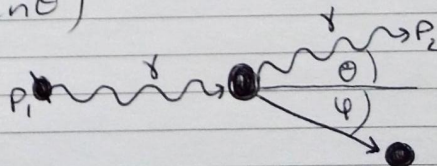
Compton Effect

momentum conservation

energy conservation

$$\begin{pmatrix} p_1 \\ 0 \end{pmatrix} = \begin{pmatrix} p_e \cos \psi \\ p_e \sin \psi \end{pmatrix} + \begin{pmatrix} p_2 \cos \theta \\ p_2 \sin \theta \end{pmatrix}$$

$$p_1 c + m_e c^2 = p_2 c + \sqrt{p_2^2 c^2 + m_e^2 c^4}$$



Square and Combine

$$p_1 p_2 (1 - \cos \theta) = m_e c (p_1 - p_2)$$

So change in wavelength:

$$\Delta \lambda = \frac{h}{p_2} - \frac{h}{p_1} = h + \frac{p_1 - p_2}{p_1 p_2} = \frac{h}{m_e c} (1 - \cos \theta).$$

Example 2 photons 1 TeV Xray. (10^{12} eV)

$$p_1 = E_1/c \rightsquigarrow \text{[wavy line]} p_2 = E_2/c$$

what energy would p_1 need for the collision to make a $e^- e^+$ pair?

momentum conserved, so if just sufficient to produce a pair: $2p_e = (p_1 - p_2)$

$$\text{and energy } (E_1 + E_2)^2 = (2E_e)^2 = (2p_e c)^2 + (2m_e c^2)^2 = (p_1 - p_2)^2 c^2 + 4m_e^2 c^4$$

hence $E_1, E_2 = m_e^2 c^4$ after lots of cancellations

$$E_1 = \frac{m_e^2 c^4}{E_2} \quad E_2 = 1 \text{ TeV.}$$

$$\approx 0.25 \text{ eV.}$$

One more: ^{Inelastic} 2 lumps of clay, rest masses $m_0 = 1 \text{ kg}$ fired at each other at $0.943c$ ($\gamma = 3$), they stick as they collide, at that moment, M_0 conserved?

$$\text{Total energy} = E = 2\gamma m_0 c^2$$

$$M_0 = 2\gamma m_0 = 6 \text{ kg.}$$

Once excess internal energy has radiated away, $M_0 = 2 \text{ kg.}$

Inelastic collisions

lecture 6

See previous example KE not conserved here, but in general Energy is always conserved

Further Consequences

1 Test: muon decay.

It Takes 30×10^6 s to reach ground^(10km), but $\frac{1}{2}$ life is 2×10^{-6} s so most shouldn't make it. Actually, more than we expect do. γ factor normally 5-10, so lifetimes stretch to $10-20 \times 10^{-6}$ s. in terms of length contraction is muon's POV, distance looks about 1-2 km

Doppler Shift

Conventional eq:

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{actual}}} = \frac{c + v_s}{c + v_r} \leftarrow \begin{array}{l} \text{velocity of source relative to medium} \\ \text{velocity of receiver} \end{array}$$

Adding Special relativity:

$$1+z = \frac{\lambda_{\text{obs}}}{\lambda_0} = \gamma \left(\frac{c+v}{c} \right) \Rightarrow \left(\frac{1 + \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = \frac{(1+\beta)^2}{\sqrt{(1-\beta)(1+\beta)}} = \boxed{\frac{1+\beta}{\sqrt{1-\beta}}}$$

This time-slaving applies whatever direction successive wavecrests are delayed. Switch signs for approaching source

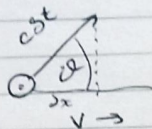
Doppler effect reduces luminosity for redshift (γ loses energy by factor $\sqrt{(1+\beta)(1-\beta)}$). Viceversa for blue shift.

Rate of Arrival varies with time dilation, by a factor of:

$$\frac{1+\beta}{1-\beta}$$

eg solar's incoming jet brighter than back jet.

Headlamp Effect



$$c \delta t \cos \theta = \delta x \quad \text{if stationary.}$$

Now Sun is moving at v :

$$\text{in our frame} \quad \cos \theta' = \frac{\delta x'}{c \delta t'} = \frac{\gamma (\delta x + v \delta t)}{\gamma (\delta t + \frac{v \delta x}{c})}$$

So we see a weird angle.

So if $\cos \theta = 0$ then
 $\cos \theta' = \beta$ in α which $= 1$.
 so $\theta' = 0$ &

$$= \frac{\delta x}{\delta t} + v$$

$$c + \frac{v \delta x}{\delta t}$$

we stationary eq.

Means rapid approach makes light source bend towards you highest intensity occurs when $\theta = 90^\circ$ ~~moving~~ so already pointing at you.

$$\cos \theta' = \frac{\cos \theta + \beta}{1 + \beta \cos \theta}$$

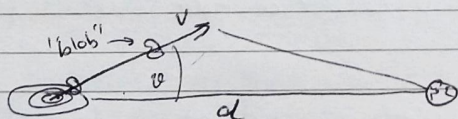
if θ' is small $\cos \theta \approx 1 - \frac{\theta^2}{2} = 1 - \frac{\theta'^2}{2} + \beta$

hence $\theta'^2 \approx \theta^2 \frac{1-\beta}{1+\beta}$ familiar!

Thus there's increase in intensity!

"Superluminal" Motion. (faster than c motion).

eg jets with both doppler boosting + headlamp effects.



if θ is small, jets appear to move at superluminal velocity

time of arrival of light to me is d/c and:

$$\text{time of arrival} = d/c \quad \text{and} \quad t_1 + (d - vt_1 \cos \theta)/c$$

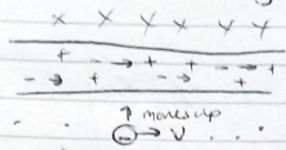
t_1 = time for something to completely transverse the jet

$$\text{tangential velocity } u = \frac{v \sin \theta}{t_1 - \frac{v}{c} t_1 \cos \theta} = \frac{\beta \sin \theta}{1 - \beta \cos \theta} c$$

apparent distance travelled tangentially = vt_1

Magnetism

eg electron moving parallel to a wire with current I at distance r .



From β^- POV it sees a +ve wire
it's attracted to wire by Coulomb force.

$$F = \frac{e^2 (\gamma - 1/\gamma)}{2\pi r \epsilon_0}$$

wire POV

$$\approx \frac{\mu_0 e^2 \gamma v^2}{2\pi r} \quad \text{since } \gamma - 1/\gamma \approx \beta^2/2 \text{ when } \beta \text{ is small}$$

and $c = \sqrt{\epsilon_0 \mu_0}$

lecture 7

Proper time

or accelerated.

time measured by stationary clock is "proper time".

$$\tau = \int \frac{dt}{\gamma(t)}$$

so $\Delta t = \gamma \Delta \tau$ ^{Proper time.}

the proper time is maximised for non accelerating observers

eg twin paradox

Stationary twin

$$\Delta T_A = \sqrt{(10 \text{ years})^2} = 10 \text{ years}$$

moving twin (0.866c)

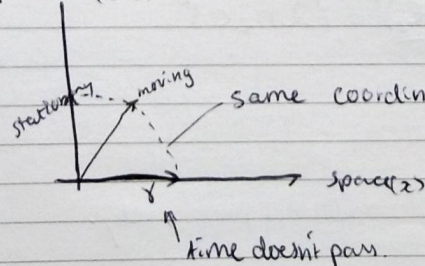
$$\Delta T_{B\&G} = \sqrt{5 \text{ yr}^2 - 4.33 \text{ yr}^2} = \sqrt{(6.25/4)} = 2.5 \text{ years}$$

5 years $\rightarrow 2 \leftarrow$

$$\times 2 \text{ journeys} = 2 \Delta T_{B\&G} = 5 \text{ years.}$$

Instead of coordinate time, let's plot proper time

proper time ($c\tau$)



As you go faster your angle with Hlt towards space axis.

4-Velocity

we can define velocity with 3 space properties & time.

4 velocity is rate of change of 4-position wrt to proper time.

$$\underline{\vec{V}} = \frac{d}{d\tau} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \gamma \frac{d}{dt} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \gamma \begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix} \quad \leftarrow \text{simplified Minkowski space.}$$

magnitude

$$\underline{\vec{V}} \cdot \underline{\vec{V}} = \gamma^2 (c^2 - v^2) = c^2 \gamma^2 \left(1 - \frac{v^2}{c^2}\right) = c^2$$

So think that we're all travelling at c , but some of us is in the time component.

4-momentum:

$$\underline{\vec{P}} = m_0 \underline{\vec{V}} = \gamma m_0 \begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix} \quad \begin{matrix} E = \gamma m_0 c^2 \\ E^2 = p^2 c^2 + m_0^2 c^4 \end{matrix}$$

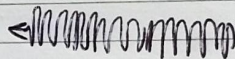
rest mass.

magnitude

$$\underline{\vec{P}} \cdot \underline{\vec{P}} = \frac{E^2}{c^2} - p^2 = (m_0 c)^2 = \left(\frac{E_0}{c}\right)^2 \quad \leftarrow \text{invariant (independent of frame).}$$

Example. remember mega ^{electron} ~~photon~~ hitting baby ^{positron} ~~photon~~

$$E_1/c = p_1$$



$$E_2/c = p_2$$

using 4 momentum:

$$\underline{\vec{P}} = \begin{pmatrix} E_1/c + E_2/c \\ E_1/c - E_2/c \end{pmatrix} \quad \begin{matrix} \leftarrow \text{time} \\ \leftarrow x \end{matrix}$$

4 momentum of $\gamma\gamma$ pair in their rest frame:

$$\underline{\vec{P}} = \begin{pmatrix} 2m_e c \\ 0 \end{pmatrix}$$

momentum is conserved, so equate before and after:

$$\frac{(E_1 + E_2)^2}{c^2} - \frac{(E_1 - E_2)^2}{c^2} = (2m_e c)^2$$

expand & simplify

$$4E_1 E_2 = 4m_e^2 c^4$$

$$E_1 E_2 = (m_e^2 c^4)$$

as found previously.

Intro to General Relativity.

Lecture 8

If a rocket accelerates $\Rightarrow$

Rocket thinks

extra velocity = dv

Earth thinks

already travelling $v' = 50\%$

$$v' + dv' = \frac{v' + dv}{1 + \frac{v' dv}{c^2}}$$

but also include time dilation:

$$1 + \frac{v' dv}{c^2}$$

$$dt' = \gamma dt$$

Hence observed acceleration:

$$a' = \frac{dv'}{dt'} = \frac{1}{dt'} \left(\frac{v' + dv}{1 + \frac{v' dv}{c^2}} \right) = \frac{1}{\gamma dt} \left(\frac{dv - v'^2 dv/c^2}{1 + \frac{v' dv}{c^2}} \right)$$

$$\text{tends to } \Rightarrow \frac{a}{\gamma^3}$$

$$= \frac{a}{\gamma} \left(\frac{\sqrt{1-\gamma^2}}{1 + \frac{v' dv}{c^2}} \right)$$

so if they're moving relativistically acceleration looks

minimal regardless.

Extra stuff

Black Holes: redshift

wavelength

$$1+z = \frac{\lambda_\infty}{\lambda_0} = \frac{1}{\sqrt{1-R_s/R}}$$

stretching γ till $z \rightarrow \infty$

seen by superfar observer.

time gets slower & slower as you get to R_s .

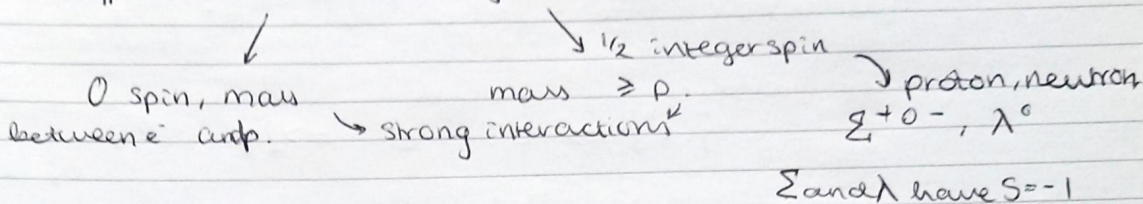
Unit 2: Elementary Particles

Lecture 1

Hadron.

2 types: meson & baryon.

Not elementary.



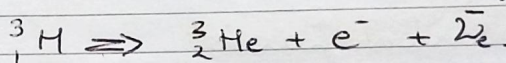
Leptons

Weak interaction. $\frac{1}{2}$ spin.

Electron (neutrino), muon (neutrino), tauon (neutrino)

Varying mass. T is $2 \times$ mass of p .

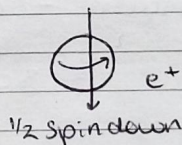
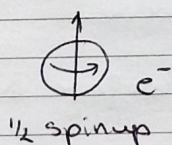
Beta decay makes one electron, but to conserve energy it must make a neutrino too.



Electrons had less energy than predicted, so neutrino was found.
now e^- and $\bar{\nu}_e$ can share energy in multiple ways.

Spin

like angular momentum. a fundamental property.



Spin is a quantum property. Don't take these pictures too literally.

$\frac{1}{2}$ spin: e^- , p , n , ν , μ , T have $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \dots$ Fermions.
These obey the Pauli Exclusion Principle.

Integer spin: Mesons, bosons. $S = 0, 1, 2 \dots$
don't obey PEP.

Pauli Exclusion: Only 2 particles can occupy the same quantum state, one spin up one spin down.

Stops white dwarves & neutron stars collapsing further.

Antimatter

Same particle but opposite charge and spin
Has a bar on the symbol except e^+ and sometimes p^- .

antimatter always produced in matter antimatter pairs

$$\text{eg: } 2\gamma \Rightarrow e^+ + e^- \Rightarrow 2\gamma \rightarrow \text{photon gets}$$

$$E_\gamma = m_e c^2$$

These get annihilated so quickly. $p_\gamma = m_e c$

Lecture 2

Strong Nuclear

$$10^{-23} \text{ s}$$

Hadrons (quarks)

many hadrons are stable

$n = 930 \text{ s}$, $p = 10^{10} \text{ yrs}$.

Weak Force

$$10^{-10} \text{ s. } \mu \text{ mean lifetime } 2.2 \times 10^{-6} \text{ s}$$

e^- are very stable

neutrinos don't decay, but can change flavour.

Conservation Laws

Energy - total rest mass of decay products $<$ initial mass of particle before decay

Linear momentum - e^- and e^+ must produce 2γ to conserve and carry momentum

Angular momentum - as above.

Charge - same before & after.

Baryon number - +1 for baryon, -1 for anti baryon, 0 for else.

Lepton number - leptons are conserved, but it needs to be same flavour eg electron number = l_e for e^- & $\bar{\nu}_e$.

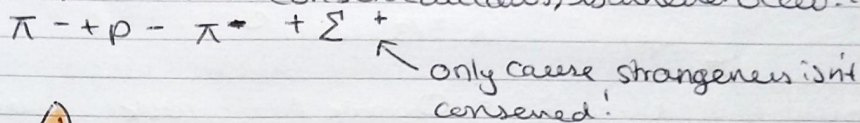
Strangeness - strangeness stays conserved.

Strange Particles.

family of particles found in 1950s that behave oddly, i.e. conserved in some strong force, but not weak.

Made in pairs, eg Kaon $K^+ + \Sigma^-$, $K^0 + \Sigma^0$, $K^0 + \Lambda^0$ all from $\pi^- + p$.

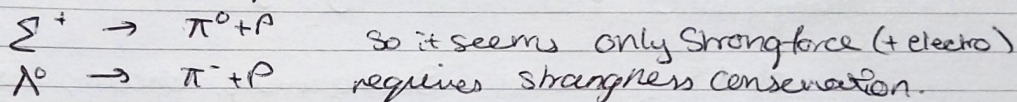
but some interactions conserve all laws, but never occur???



However 

Sometimes a strange particle decays, & strangeness ISN'T conserved.

Particles prefer strong decay, but sometimes strong doesn't allow decay, so they need weak decay:



Strong force - conserved
Weak force - may change by ± 1

Example: $K^0 \rightarrow \pi^+ + \pi^-$
 $S = +1 \quad 0 \quad 0$ so change of 1,
hence could interact weakly

$\Sigma^0 \rightarrow \Lambda^0 + \gamma$
 $S = -1 \quad -1 \quad 0$ There's a γ , so
most likely electromagnetic. However
strangeness is conserved, so it prefers
strong decay

$\Xi^0 \rightarrow \pi^0 + n$
 $S = -2 \quad 0 \quad 0$ Impossible!

lecture 3

Quarks

1963 discovery.

$\frac{1}{2}$ Spin, and third charges. All have

baryon number	charge	strangeness	charm	beauty	truth	Mass
6 types						
up	$+\frac{2}{3}$	0	0	0	0	335 MeV
down	$-\frac{1}{3}$	0	0	0	0	338 "
strange	$-\frac{1}{3}$	-1	0	0	0	510 "
charm	$+\frac{2}{3}$	0	1	0	0	1500 "
bottom	$-\frac{1}{3}$	0	0	-1	0	4750 "
top	$+\frac{2}{3}$	0	0	0	1	174 GeV.

too unstable for hadrons

Mesons = Quark Antiquark
Baryon = 3 Quark
Antibaryon = 3 Antiquarks

$u\bar{d}$
 uud
 $\bar{u}\bar{d}\bar{d}$

Examples:

uud
Baryon number: 1
Charge: +1
Strangeness: 0
} proton

udd
1
0
0
} neutron

uus
1
+1
-1
} Σ^+

$d\bar{d}s$
1
-1
-1
} Σ^-

Strangeness, charm, beauty, truth are conserved in electromagnetic force, but not in weak force.

Quark Spin

Let's say in a meson: Since all quarks have $\frac{1}{2}$ spin:

$\uparrow\uparrow$ Spin 1 $\frac{1}{2} + \frac{1}{2}$ and $\uparrow\downarrow$ Spin 0 $\frac{1}{2} - \frac{1}{2}$ or $\downarrow\downarrow$ Spin -1

In a baryon with uud

$\uparrow\uparrow\uparrow$ Spin $\frac{3}{2}$
 $\approx \Delta^+$ particle

$\uparrow\downarrow\downarrow$ Spin $\frac{1}{2}$
 $\approx$ Proton

$\uparrow\uparrow\downarrow$ still spin $\frac{1}{2}$
 $\approx$ Proton too!

You'd think uud would always mean a proton. wow!

Quark Colour

Σ^- is a baryon SSS , $S=3$ $spin=3/2$. Hence

$\uparrow \uparrow \uparrow$

But Pauli Ex forbids same state same particles. Identity! This can work if the quarks have different colourie RAB.

So in baryons (+ antibaryons) colour is arranged so they're colourless. Baryons = RAB = white.
Antibaryon: Magenta, cyan, yellow = white.

In meson = red + antired = white.

Quark Confinement

Quarks aren't naturally isolated. ever. Impossible. Because of it, we infer properties based on systems, and we can't accurately know mass.

Lecture 4

Fundamental Forces

A particle experiences/interacts with a boson if it's charged in that property of the boson.

if we put energy into a system like accelerating a particle, virtual photons get shaken off and become real. All of the bosons can be described this way.

Weak force

- Quarks, leptons
- W^+ , W^- , Z^0 bosons
- the "charge" they mediate is flavour.
- Quantum Electrodynamics
- spin 1
- charges ± 1 for W^+/W^-
0 charge for Z

Strong force

- Quarks, hadrons
- gluon, never observed!
- the "charge" is colour
- Quantum Chromodynamics
- fundamental strong = binds quarks
- residual strong = attracts proton in nucleus.
- spin 1
- charge 0

Photon
γ
gluon
g
boson
Z^0
W^+, W^-
W

Gravity

- maybe the graviton?
- the "charge" is mass
- gravitons massless
- Spin 2
- Hypothetical,
- 0 charge.

Electromagnetic

- many particles (electrically charged).
- involves photons γ
- photons "massless"
- Spin 1
- 0 charge

Electroweak Theory.

Bring electromagnetic + weak together. At high energies, $> 100 \text{ GeV}$ electroweak would be carried by W^+, W^-, W^0 and B^0 bosons. W^0, B^0 can't be observed directly

The $W^0 + B^0$ combine $\Rightarrow Z^0$ bosons or γ .

At ordinary energies, the symmetry is broken, and electroweak separates.

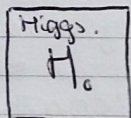
Higgs Boson

For electroweak to work, we need a Higgs Boson.

The Higgs Field breaks the symmetry. The Boson was found at CERN w/ $126 \text{ GeV}/c^2$.

Higgs

- Symbol H_0 (not Hubble!)
- no charge, no colour
- Spin 0.



Higgs field is an energy field. It gives rest mass to particles. It doesn't create mass, but "assigns".

Process of giving mass = Higgs effect.

If the field didn't exist, particles have no mass, no gravitational attraction, outtravelling etc.

Quarks interact strongly w/ field, leptons interact slightly less, γ don't interact.

The Standard Model

Combination of Quark theory, electroweak and QCD make the Standard Model.

All matter is made of Quarks & leptons, all particles have anti-particle, and every force is due to 4 basic interactions.

Standard model has problems e.g. gravity.

Grand Unified Theory

Attempt to unify everything, gravity, CK etc...

One idea is Quarks & leptons are two aspects of a single class of particle. Early universe idea.

Another idea is that protons decay, in standard model this doesn't occur.

The Big Bang

- All fundamental forces unified.
- at 10^{-43} s / 10^{32} K / 10^{19} GeV Gravity condenses out
- at 10^{-27} s / 10^{27} K / Strong force condenses out.
- at 10^{-6} s / 10^{13} K most hadrons from destroyed.
- at 10^{-4} s antimatter annihilation creates γ and leptons Quarks start to bind
- at 10 s, e^-e^+ annihilation removes e^+ . dominated by γ and n
- minutes in, H^2 , H' , He fuses.
- 300'000 yrs. mass domination. atoms created.
2.7 K era, end of opaque universe

matter wins out
over antimatter

lecture 5

The next mass of decaying particles must ALWAYS be more than the products eg $\Sigma \rightarrow n + K^-$ cannot occur!

Examples:

1) i) $\mu^+ + \mu^- \rightarrow e^+ + e^-$

L_μ	1	-1	$\rightarrow$	0	✓
L_e		0	$\rightarrow$	-1 + 1	✓
Q	1	-1	$\rightarrow$	1 - 1	✓

possible!

ii) $\tau^- \rightarrow e^- + \nu_e$

L_τ	1	$\rightarrow$	0	x
L_e	0	$\rightarrow$	1 + 1	x

Not possible!

- 2) a) Meson w/ $S=-1$ $Q=1$ not possible
 b) Baryon w/ $S=-1$ $Q=-1$ odd = Σ^-
 c) Baryon w/ $S=-3$ $Q=-1$ $sss = \Omega^-$ not possible (Particle)
 d) Hadron w/ $S=0$ $Q=0$ neutron odd, $u\bar{u}, d\bar{d}, s\bar{s} \dots$
 e) Hadron w/ $Q=3$ not possible

$\bar{S}$ strangeness = 1

lecture 6

Particle Accelerators & Colliders.

Particle Accelerators, needed to get charged particles close together. High energies overcome electrostatic repulsion.
 a) Shortens particle de Broglie wavelength. Makes higher resolution results.

$$\text{momentum } p = \frac{h}{\lambda} \quad p = \frac{h}{mv} \quad \lambda = \frac{hc}{E_k}$$

Tevatron Fermilab uses 1 TeV, wavelength $\approx 1.24 \times 10^{-12}$ m.

Feedback Matt calls it de Brog-lee, it's actually de "Broy".

First accelerators use constant P.D to accelerate protons.

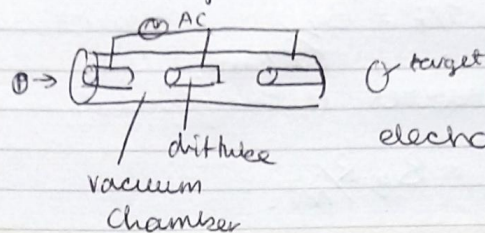
$$V_{\text{output}} = 2 N V_P$$

$\uparrow$
of stages $\uparrow$ Peak voltage.

Many design flaws, couldn't go over $\sim$ MeV otherwise hardware breakdown eg Insulators.

Linear Accelerator LINAC

beam travels through hollow electrodes. in AC supply
As they cross through electrodes, they're accelerated by E field



$t/2$ = half period of AC voltage.

length of drift tube of particles speed $v = L_n = vt/2$

$$KE_n \text{ of particles} = neV_0 = mv^2/2$$

$n = \text{gaps}$ $\uparrow$
voltage.

$$L_n = neV_0/2m)^{1/2} t. \quad \text{so } L_n \text{ must increase as } n^{1/2}$$

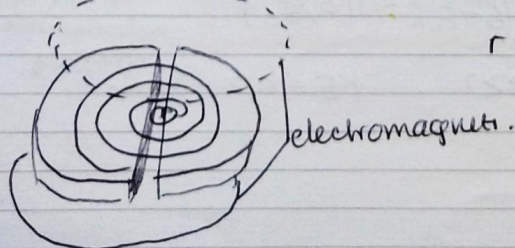
$36 \rightarrow$ $\rightarrow p.$ but if particles speedy, L_n is roughly the same throughout.

cheap, don't need big magnets, no high voltages.

High energies need very long LINACS. KE is hence very limited.

Cyclotrons

Particles start in centre. B field bends path in spiral.
2 metal semicircular chambers, called Dees, connected to AC Voltage. feels no electric field in Dee, but in the gap between there's an accelerating voltage again in KE



$$F = BqV = mv^2/r$$

$$r = mv/Bq = p/Bq. \quad \text{increases } v, \text{ so radius increases.}$$

one rotation takes $t = 2\pi r/v$

$$t = 2m\pi/Bq \quad \text{not dependent on speed!}$$

$$\text{frequency } \nu = 1/t = Bq/2\pi m \quad \text{cyclotron frequency.}$$

the velocity increases as it spirals.

$$V_{\max} = Bq R/m.$$

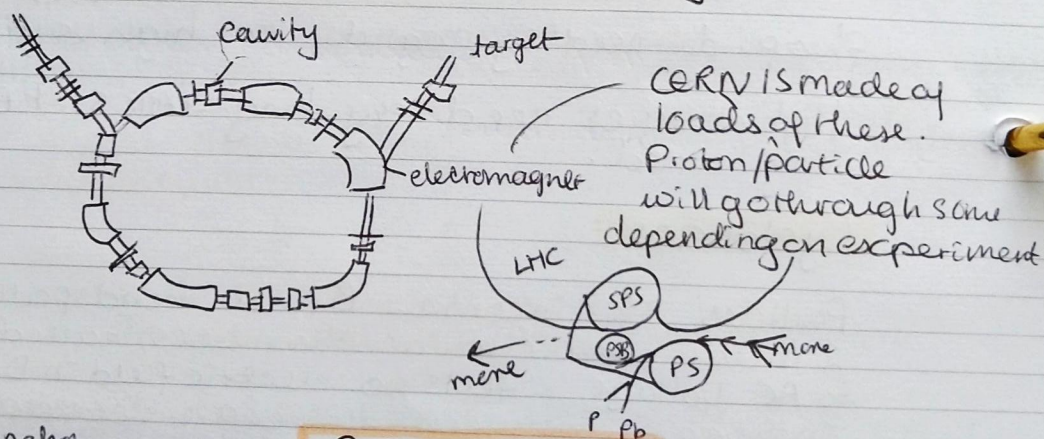
$$KE_{\max} = \frac{1}{2} m V_{\max}^2 = q^2 B^2 R^2 / 2m.$$

Small, useful for medical stuff.

High velocity makes particle mass to increase. makes radius larger. Charges start to arrive too late to be accelerated across gap. Hence 1 GeV is limit.

Synchrotrons

Rings split into sections. B field increases as particles move through ring multiple times.



Synchrotron radiation
~~not energy of proton~~

$$P = \frac{e^2 c}{6\pi\epsilon_0 R^2} (E/E_0)^4$$

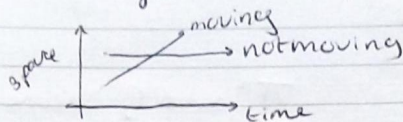
radius R
 relativistic energy E.

If for a particle is $\gamma \gg 1$, $KE = cp = BeRc$.

lecture 7

Feynman Diagrams

2D image plotted on spacetime, sometimes time x space.



particle key:

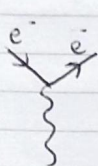
→ matter particle

~~~~~ photon

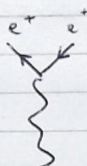
ooooo gluon

-----  $Z^0, W^+, W^-$  boson

← antimatter

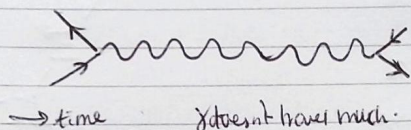


$e^-$  emits  $\gamma$  and keeps going



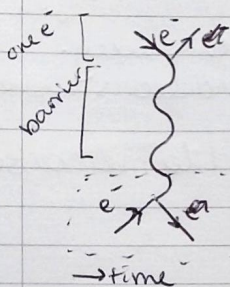
$e^+$  absorbs  $\gamma$  and keeps going

## $e^+ e^-$ annihilation & "creation"



it doesn't have much

Although they look like they're hitting at an angle, but they're hitting head on.

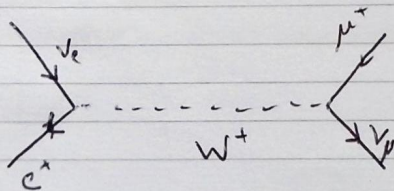


2 electrons repelling each other.

So electro magnetic force keeps them apart. Hence the diagram orientation.

same different  $e^-$

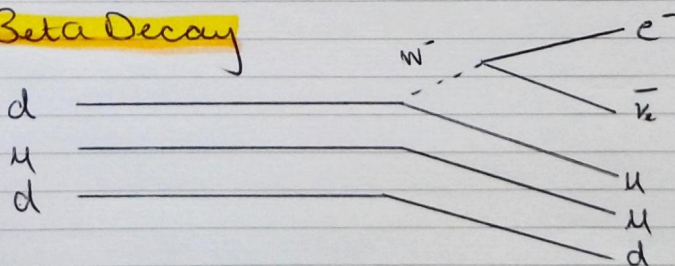
Virtual photon.



Numbers conserved  
+ve left  $\rightarrow W^+ \rightarrow +ve$  right.

allowed

## Beta Decay



neutron  $\rightarrow$  proton.

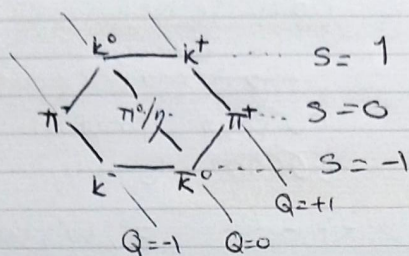
$n \rightarrow p + e^- + \bar{\nu}_e$

# Lecture 8

## The Eightfold Way

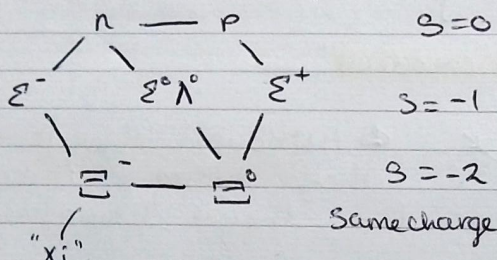
A way to organise hadrons and predict them.

Here is the meson octet

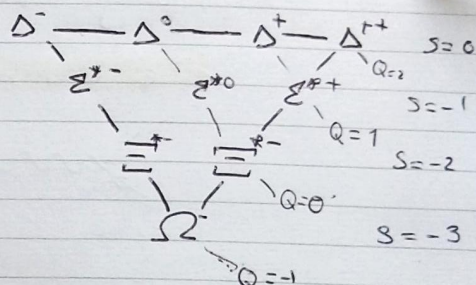


$\pi^0$  and  $\eta$  share a place.  
There's also  $\eta'$  eta prime,  $Q=0, S=0$   
but the diagram is a bit stupid

Here's the Baryon Octet  
spin  $\frac{1}{2}$  particles



decuplet (spin  $\frac{3}{2}$ )



The  $\Omega^-$  (sss) was predicted this way.  
These paths occur naturally if all hadrons were made  
of only u, d, s.

You don't need to memorise these!